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THE EFFECT OF CLIMATIC CHANGES UPON A MULTIPLE-SPAN REINFORCED CONCRETE ARCH BRIDGE

BY

WILBUR M. WILSON

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THE EFFECT OF CLIMATIC CHANGES UPON A MULTIPLE-SPAN REINFORCED CONCRETE ARCH BRIDGE

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CONTENTS

LIST OF FIGURES

LIST OF FIGURES (Continued)

LIST OF TABLES

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THE EFFECT OF CLIMATIC CHANGES UPON A MULTIPLE-SPAN REINFORCED CONCRETE ARCH BRIDGE

I. Introduction

1. *Object of Tests.*—Variations in temperature and in the moisture content cause concrete to change in volume, and these variations produce changes in the shape of an arch rib that are accompanied by direct and bending stresses in the rib and by an overturning thrust upon the piers. The object of this investigation was to observe the change that took place in a multiple-span arch bridge in order that the magnitude of the changes might be determined, and in order that the measured and the computed changes might be compared.

2. *Acknowledgments.*—This investigation was made as a part of the work of the Engineering Experiment Station of the University of Illinois, of which Dean M. S. Ketchum is the director, and of the Department of Civil Engineering, of which Professor W. C. Huntington is the head. It also constitutes a part of the program of the Committee on Concrete and Reinforced Concrete Arches of the American Society of Civil Engineers.* Most of the work was done by members of the Staff of the Engineering Experiment Station, but the expenses, including some of the salaries, were paid from funds provided by the American Society of Civil Engineers.

Messrs. George E. Keranen and J. M. Hardesty, Graduate Research Assistants of the Engineering Experiment Station, and Cyrus Fishburn, a special assistant employed by the Committee, are to be specially commended for their persistence and painstaking efforts in securing accurate data in the face of physical discomfort. Messrs. A. H. Sorenson and E. C. Grafton, Graduate Research Assistants provided by the Arch Committee of the A. S. C. E., made the analysis of the arches presented in Figs. 4 and 7.

II. Description of Test

3. *Description of Bridge.*—The bridge† on which the observations were made is the six-span, two-rib highway bridge over the Vermilion River at Gilbert Street, Danville, Illinois. The general dimensions of

*The members of this Committee are C. T. Morris, Chairman, E. H. Harder, A. C. Janni, George E. Beggs, and W. M. Wilson.

†The bridge was designed by Harrington, Howard, and Ash, Consulting Engineers of Kansas City, Mo.

the bridge and the type of structure are shown in Figs. 1 and 4. It was completed in 1922.

The bridge is of the open-spandrel type except that, for the middle three panels of each span, the rib is connected to the deck by means of a spandrel wall. There are expansion joints in the deck adjacent to the piers and at the extremities of the saddle, for spans 2, 3, 4, and 5, and adjacent to the piers only for spans 1 and 6. The piers rest upon a good grade of shale.

The computed pier reactions due to dead load are given in Fig. 7, and the position and magnitude of the computed temperature thrust are given in Fig. 4. The computations for these reactions are based upon the assumption that the piers are fixed and inelastic and that the rib is not restrained by the deck.

The observations included the rotation of the piers and the change in the width of the expansion joints for all spans and, for the west rib of span 2, they included the change in the temperature of the concrete, the rise and fall of points on the arch rib, the rotation of points on the rib, the transverse deformation of the concrete, and the longitudinal deformation of the steel and concrete. Observations were made on eighteen different days distributed over a period of approximately 20 months, including at least one extremely hot day each summer and one extremely cold day each winter.

4. *Changes in Temperature of Arch Rib.*—The temperature of the concrete in the west rib of span 2 was measured with mercury thermometers inserted in temperature wells. A well consisted of a hole in the concrete about an inch in diameter in which two corks were inserted, one about an inch from the bottom and the other at the top of the hole. The portion of the well below the lower cork was filled with cup grease. There was a hole at the center of each cork just large enough so that a thermometer forced into it would be held firmly in place. When a thermometer was in a well its bulb was in contact with the cup grease, a good conductor of heat that had already attained the temperature of the surrounding concrete. Cup grease was used in preference to a liquid because it will stay in place in a well on the underside of the rib. The dead stems of all thermometers were so long that the thermometers could be read without being removed from the wells.

The temperature was observed at mid-depth of the rib and at the reinforcing steel, a depth of approximately 3 inches. The temperature at mid-depth was taken at a section just below N2, Fig. 1, where the rib is approximately 40 inches thick, and just below N4 where it is approximately 30 inches thick.

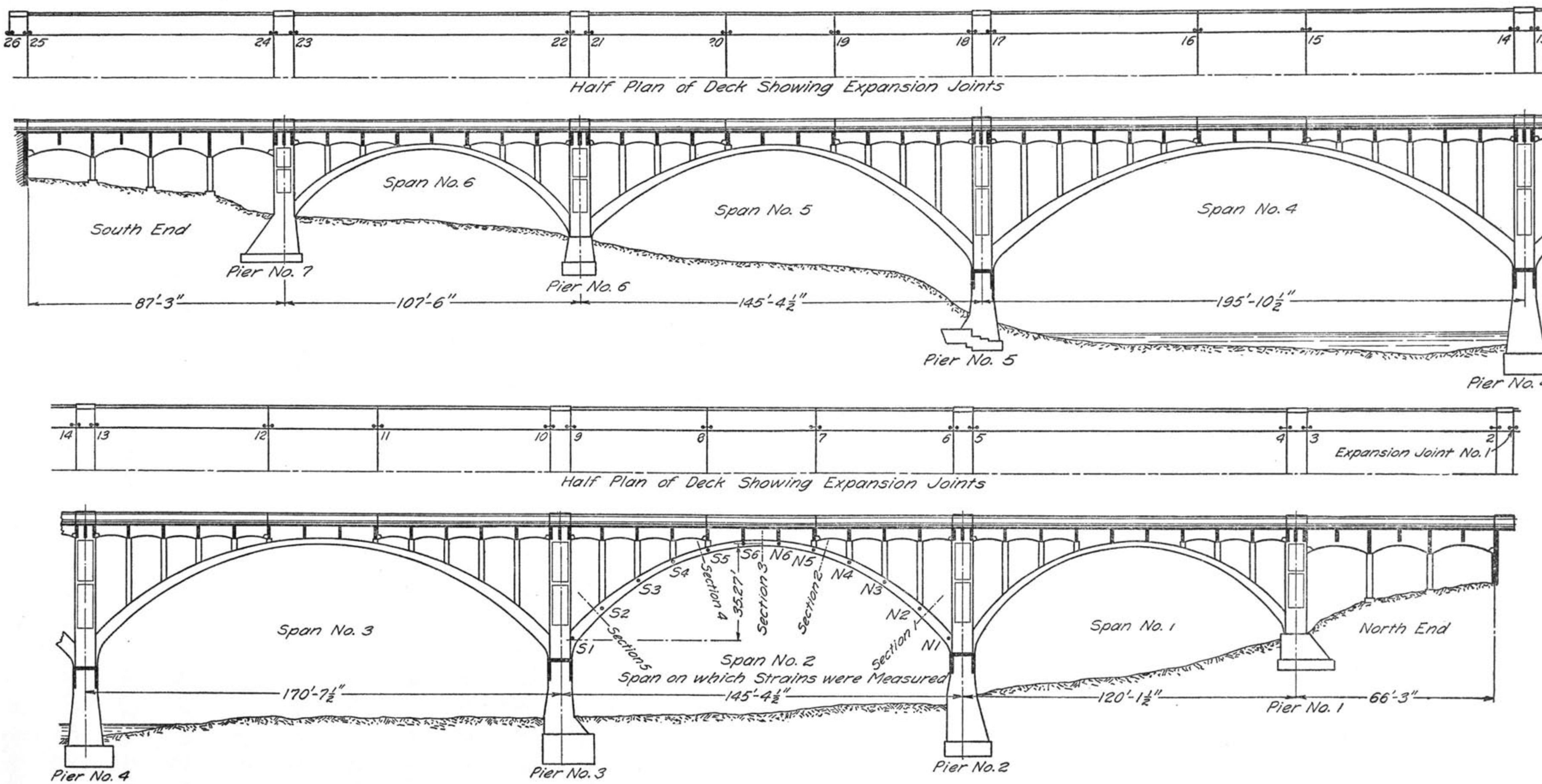

Fig. 1. Longitudinal Section of Bridge

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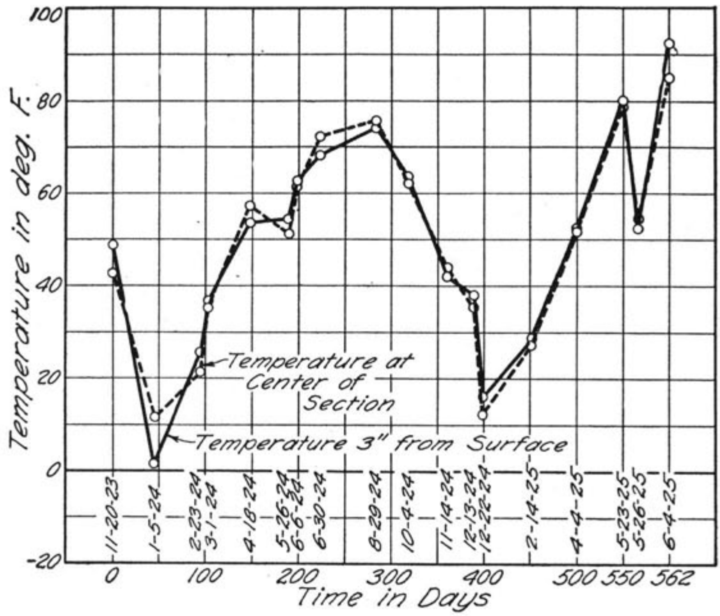


FIG. 2. VARIATION IN TEMPERATURE OF CONCRETE IN ARCH RIB

The thermometer wells for obtaining the temperature adjacent to the reinforcing steel were located at the middle of the strain-gage lines for one of the inside and for each of the two outside bars at sections 1, 2, 4, and 5, and for each of the two outside bars only, at section 3. These wells were located in both the top and the bottom of the rib. Wells for obtaining the temperature at a depth of 3 inches from the vertical surfaces were located on the east and west faces of the rib at sections 1, 2, 4, and 5. The temperature at a given section was taken just before or just after the strain gage had been read at that section.

The temperature in the concrete is presented graphically in Fig. 2, the broken line representing the temperature at the center of the section, and the full line that at a depth of 3 inches. The temperature at a point 3 inches from the surface is, for most days, the average of the readings in thirty-six wells; there were a few days when the temperature on the vertical faces of the rib was not read. For these days, only 28 readings were included in the average; and on January 5, 1924, the temperature was read in only 18 of the 3-inch wells. The temperature at the center of the section is the average of two readings, one taken where the thickness of the rib is 40 inches and the other

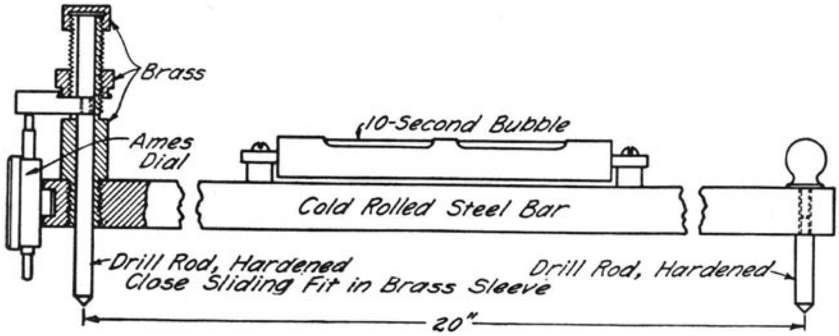


FIG. 3. LEVELBAR

where it is 30 inches. On all but two days most of the thermometers were read in the forenoon, when the rib was shaded; but on November 20, 1923, and June 4, 1925, they were read in the afternoon.

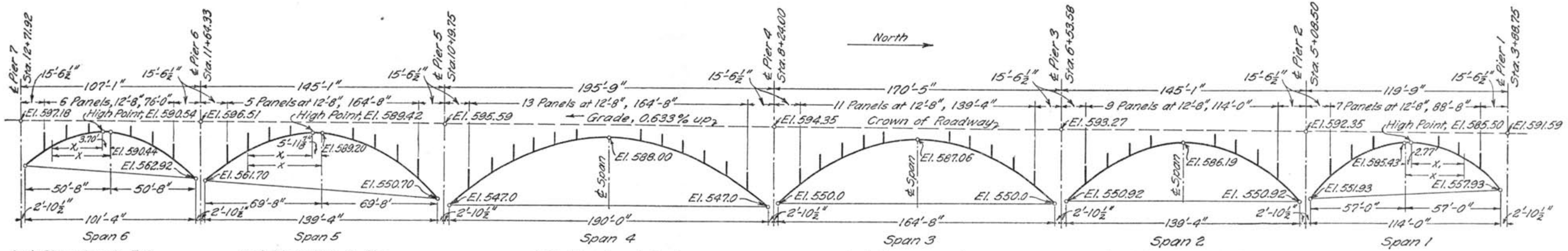
5. *Rotation of Piers and Points on Arch Rib.*—The rotation of the piers and of points on the west rib of span 2 was measured with a levelbar.* This instrument is illustrated in Fig. 3.

In order to use the levelbar, two square pins are inserted in the body whose rotation is to be measured. These pins are at the same level and are spaced 20 inches apart with the top surfaces horizontal. One leg of the levelbar rests in a small round hole in one pin and the other leg rests in a slot in the second. In taking readings, the instrument is set on the pins and the nut is turned until the bubble is in its mid position and the reading of the dial is recorded. The instrument is then turned end for end and a second reading is taken. The difference between the two readings is twice the difference in level of the two pins. The change in the difference in level of the two pins divided by the distance between them, is the rotation of the body.

The dial on the levelbar is graduated to 0.001 inch and fractions of a division are estimated with the eye, so that readings are recorded to 0.0001 inch. Duplicate readings were taken in all cases and the two sets of readings usually agreed within 0.0004 inch. A levelbar carefully manipulated should show the angular position of a body to within 0.00001 radian or 2.06 seconds.

The rotation of the piers was observed near the bottom at the surface of the ground and near the top at the springing for piers 2, 3, and 5, and at the springing only for piers 1, 4, and 6. The rotation

*The levelbar was developed by the Joint Committee on Stresses in Railroad Track.



Span 6 — Arch Dimensions in Feet

x	Right Half y	Right Half t	Left Half y	Left Half t
0	0	1.67	0	1.67
3.70			-0.10	1.67
6.33	+0.79	1.68	0.0	1.67
12.67	2.36	1.71	+0.73	1.68
19.00	4.58	1.78	2.17	1.71
25.33	7.60	1.89	4.28	1.79
31.67	11.29	2.08	7.17	1.95
38.00	15.88	2.34	10.83	2.10
44.33	21.27	2.70	15.29	2.53
50.67	27.52	3.17	20.52	3.00

Section through Arch Rib Same as for Span 1

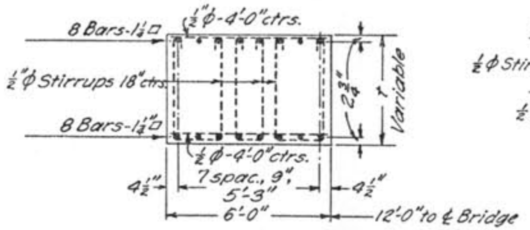
Span 5 — Arch Dimensions in Feet

x	Right Half y	Right Half t	Left Half y	Left Half t
0	0		0	
5.95			-0.24	2.33
6.33	+0.56	2.34	-0.23	2.33
12.67	1.68		+0.03	
19.00	3.38	2.41	0.84	2.35
25.33	5.75		2.23	
31.67	8.63	2.57	4.21	2.44
38.00	12.11		6.67	
44.33	16.11	2.90	9.68	2.70
50.67	20.70		13.23	
57.00	26.02	3.44	17.37	3.19
63.33	32.11		22.15	
69.67	38.50	4.25	27.50	4.00

Section through Arch Rib Same as for Span 2

Span 4 — Arch Dimensions in Feet

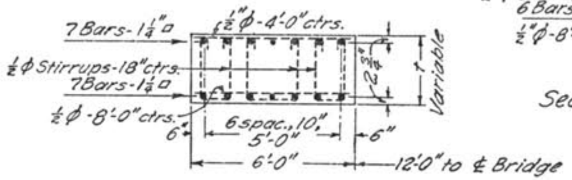
x	y	t
0	0	3.00
6.33	0.17	3.00
19.00	1.51	3.02
31.67	4.26	3.09
44.33	8.42	3.24
57.00	14.04	3.51
69.67	21.30	3.93
82.33	30.20	4.53
95.00	41.00	5.33



Section through Arch Rib

Span 3 — Arch Dimensions in Feet

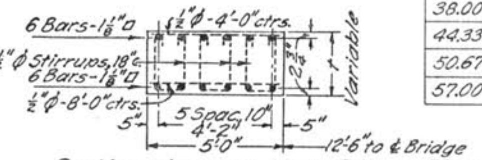
x	y	t
0	0	2.50
6.33	0.20	2.51
19.00	1.82	2.52
31.67	5.13	2.61
44.33	10.17	2.80
57.00	17.06	3.14
69.67	26.02	3.67
82.33	37.06	4.42



Section through Arch Rib

Span 2 — Arch Dimensions in Feet

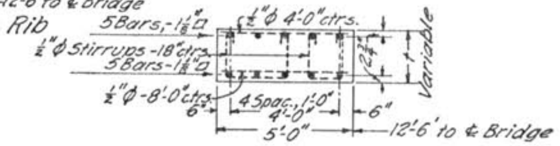
x	y	t
0	0	2.25
6.33	0.27	2.25
19.00	2.44	2.28
31.67	6.88	2.42
44.33	13.66	2.70
57.00	23.03	3.21
69.67	35.27	4.00



Section through Arch Rib

Span 1 — Arch Dimensions in Feet

x	Right Half y	Right Half t	Left Half y	Left Half t
0	0	1.92	0	1.92
2.77	-0.07	1.92		
6.33	+0.05	1.92	+0.58	1.92
12.67	0.72	1.93	1.88	1.94
19.00	2.19	1.95	3.95	1.99
25.33	4.40	2.02	6.82	2.07
31.67	7.46	2.12	10.47	2.20
38.00	11.23	2.28	14.94	2.39
44.33	15.83	2.52	20.25	2.65
50.67	21.22	2.84	26.42	2.98
57.00	27.50	3.25	33.50	3.42



Section through Arch Rib

Thrust Due to a One-Degree Fahrenheit Rise in Temperature of the Concrete. ($w = 0.0000049$)

Span No.	Thrust, lb.	Vert. Distance, Axis of Rib at ℄ of Span to Thrust Line, in.	Angle Thrust Line Makes with the Horizontal
1	111.35	86.7	2°-24'-29"
2	145.66	97.9	0
3	227.63	101.9	0
4	329.16	111.7	0
5	185.88	91.0	3°-43'-00"
6	138.32	65.5	3°-18'-25"

Fig. 4. Temperature Thrust

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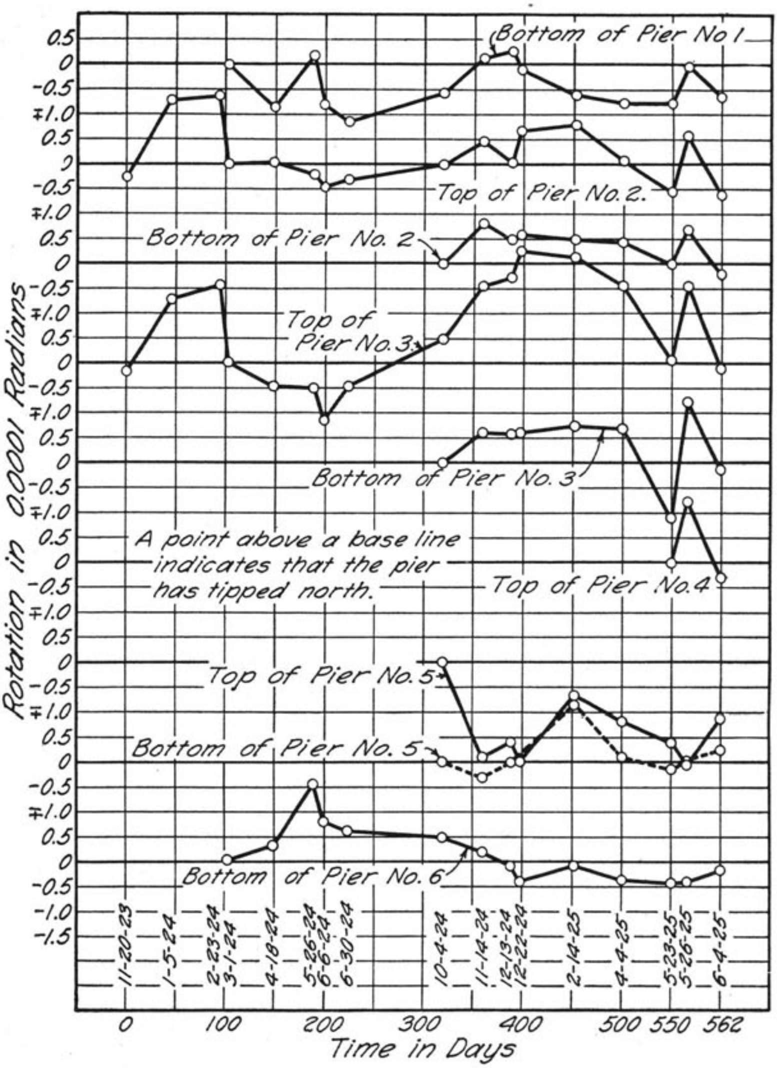


FIG. 5. ROTATION OF PIERS

of the rib was measured at points N1 to N6 and S6 to S1. The locations of the piers and the points on the rib are shown in Fig. 1.

The rotation of the piers is presented graphically in Fig. 5 and that of points on the arch rib in Fig. 6. These movements are discussed in Section 14.

6. *Rise and Fall of Points on Arch Rib.*—The rise and fall of points on the west rib of span 2 were measured at the points N1 to N6

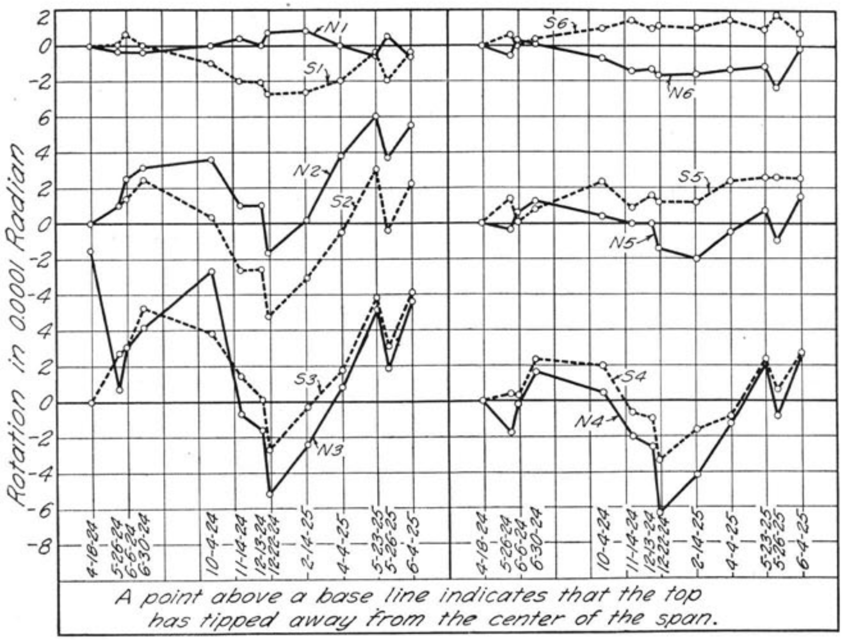


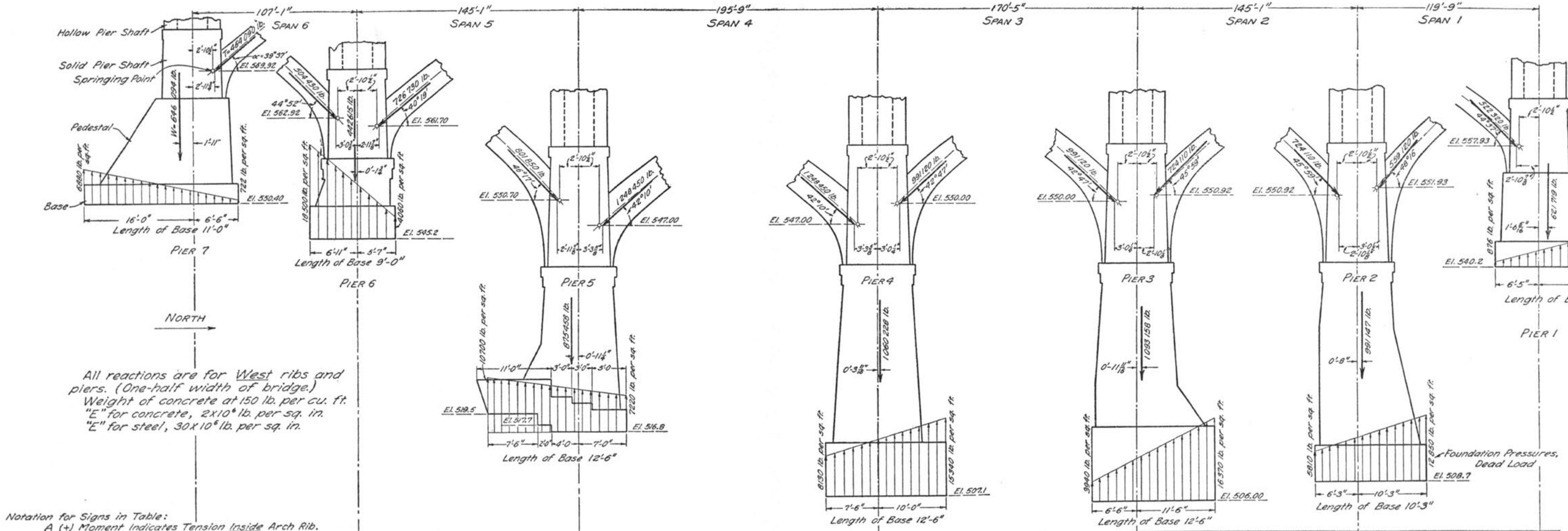
FIG. 6. ROTATION OF POINTS ON ARCH RIB

and S1 to S6. The following method was used to measure this movement:

Steel pins were embedded in the face of the rib at each point where a level reading was to be taken, and an invar steel tape having a ring at its upper and a weight at its lower end, was suspended from the pin. A level rod was attached to the tape just above the weight, so that the latter held the rod in a vertical position in addition to maintaining a constant tension on the tape. The elevation of a pin was determined from the readings of the rod taken with a wye level set upon the ground. A set of readings was taken with the level near the north pier and a set of check readings with it near the south pier of the span. With very few exceptions, the two sets of readings checked within 0.003 foot.

No correction was made for the change in length of the tape due to changes in its temperature, since the range of temperature encountered would cause a change in the length of the invar tape of only about 0.002 foot.

The observed movements are presented in Fig. 8, all displacements being reckoned from the position of the rib on March 1, 1924.



Notation for Signs in Table:
A (+) Moment Indicates Tension *Inside* Arch Rib.

Dead Load Reactions at Springing Corrected for Rib Shortening	Moment	M	-359 078 in.lb.	-672 017 in.lb.	-661 300 in.lb.	-763 233 in.lb.	-4 321 700 in.lb.	-4 321 700 in.lb.	-1 180 499 in.lb.	-1 180 499 in.lb.	+173 050 in.lb.	+173 050 in.lb.	-848 949 in.lb.	-149 393 in.lb.
Thrust	Total	T	464 086 lb.	504 432 lb.	726 733 lb.	801 855 lb.	1 248 450 lb.	1 248 450 lb.	991 121 lb.	991 121 lb.	724 109 lb.	724 109 lb.	559 117 lb.	522 322 lb.
Thrust	Angle with H.	α	39° 37′	44°-52′	40° 19′	46°17′	42° 10′	42°10′	42°47′	42°47	45°59′	45° 59′	48°16′	44°37′
Thrust	Horizontal Comp.	H	357 457 lb.	357 457 lb.	554 139 lb.	554 139 lb.	924 650 lb.	924 650 lb.	727 324 lb.	727 324 lb.	508 210 lb.	508 210 lb.	371 769 lb.	371 769 lb.
Thrust	Vertical Comp.	V	295 973 lb.	355 916 lb.	470 182 lb.	579 569 lb.	838 840 lb.	838 840 lb.	673 271 lb.	673 271 lb.	515 806 lb.	515 806 lb.	417 612 lb.	366 889 lb.
Weight of Pier, lb		W	646 094	442 615		875 458		1 060 228		1 093 158		991 147		621 719
Reactions at Springing Points Due to Abutment Displacement	1.- If Right-hand Abutment Rotates 0.001 Radian in a Clockwise Direction:	M	-942 680 in.lb	-2 037 701 in.lb.	-1 745 623 in.lb.	-3 832 685 in.lb.	-2 892 041 in.lb.	-6 476 201 in.lb.	-1 869 284 in.lb.	-4 212 820 in.lb.	-1 272 412 in.lb.	-2 875 860 in.lb.	-739 029 in.lb.	-1 783 182 in.lb.
		H	← 6 152 lb.	6 152 lb. →	← 8 377 lb.	8 377 lb. →	← 11 205 lb.	11 205 lb. →	← 8 059 lb.	8 059 lb. →	← 5 784 lb.	5 784 lb. →	← 4 043 lb.	4 043 lb. →
		V	↓ 476 lb.	476 lb. ↑	↓ 587 lb.	587 lb. ↑	↓ 1 572 lb.	1 572 lb. ↑	↓ 1 186 lb.	1 186 lb. ↑	↓ 959 lb.	959 lb. ↑	↓ 976 lb.	976 lb. ↑
	2.- If the Length of the Span Increases 1 Inch without the Piers Rotating:	M	-4 958 221 in.lb.	-5 368 608 in.lb.	-6 709 370 in.lb.	-7 241 970 in.lb.	-11 204 190 in.lb.	-11 204 190 in.lb.	8 058 885 in.lb.	-8 058 885 in.lb.	-5 784 190 in.lb.	-5 784 190 in.lb.	-4 790 493 in.lb.	-4 480 063 in.lb.
		H	← 23 177 lb.	23 177 lb. →	← 22 641 lb.	22 641 lb. →	← 29 463 lb.	29 463 lb. →	← 23 509 lb	23 509 lb. →	← 17 780	17 780 lb. →	← 16 597 lb.	16 597 lb. →
		V	↑ 1263 lb.	1 263 lb. ↓	↑ 1 388 lb.	1 388 lb. ↓	0	0	0	0	0	0	↓ 647 lb	647 lb. ↑

Fig. 7. Pier Reactions

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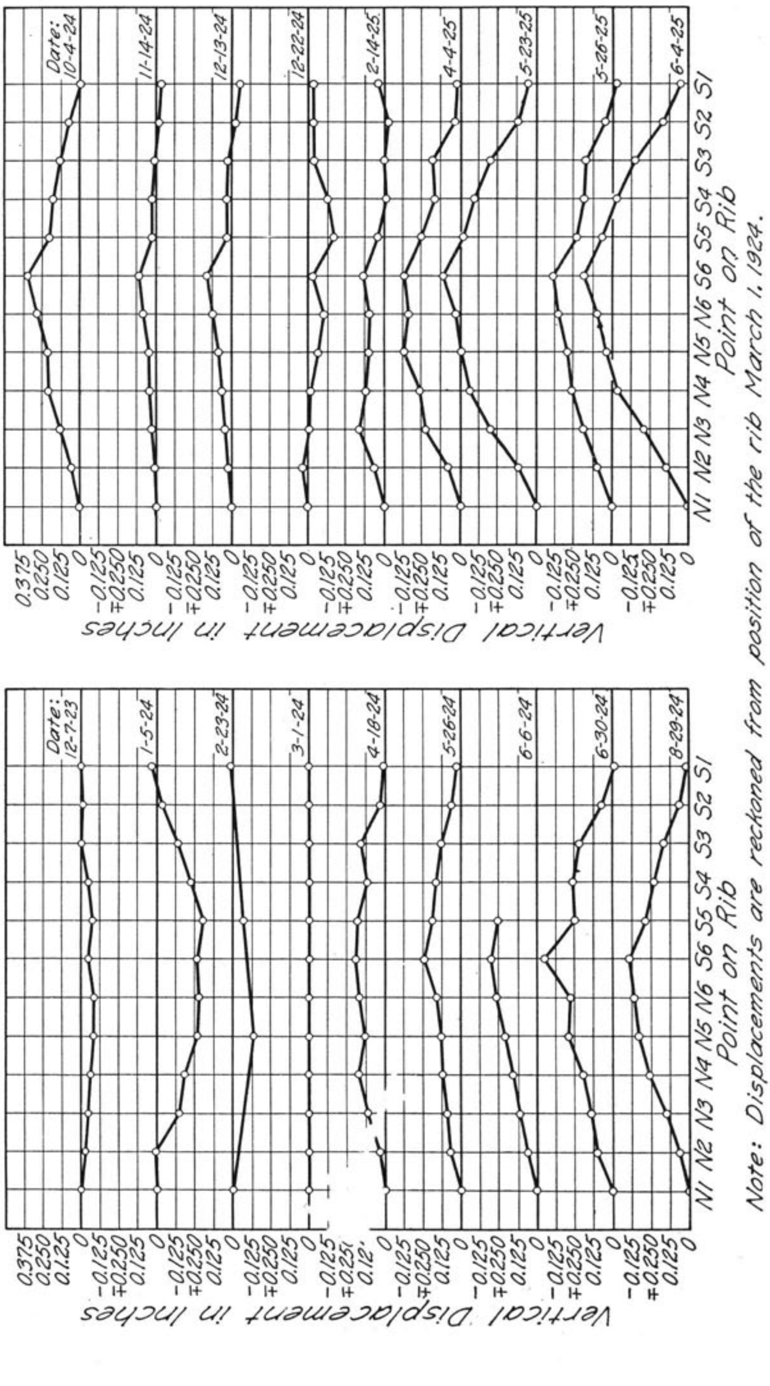

FIG. 8. VERTICAL DISPLACEMENT OF POINTS ON ARCH RIB

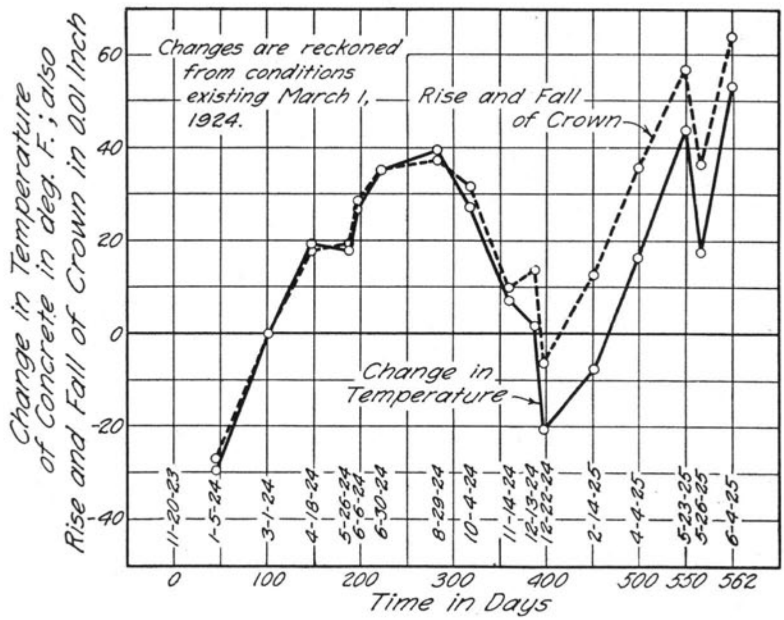

FIG. 9. RELATION BETWEEN RISE AND FALL OF CROWN AND CHANGE IN TEMPERATURE OF CONCRETE

The relation between the rise and fall of the crown and the change in the temperature of the concrete is shown in Fig. 9. The ordinates of the broken line represent the average of the movement of the two middle points, N6 and S6; and the ordinates of the full line represent the change in the temperature of the concrete, the value used being the mean of the temperatures at the center and at points 3 inches from the surface of the rib. These results are discussed in Section 15.

7. *Deformation of Concrete and Steel.*—The longitudinal deformation of the rib was measured with a 24-inch strain gage at sections 1, 2, 3, 4, and 5, shown in Fig. 1, page 8. The deformation of the steel was read on one of the inside and on both of the two outside bars, on both the top and the bottom of the rib, at sections 1, 2, 4, and 5, and on the two outside bars, on both the top and the bottom of the rib, at section 3. The longitudinal deformation of the concrete was read on gage lines located, one ten inches from the west edge, and the other ten inches from the east edge of the rib, on both the top and the bottom of the rib, at sections 1, 2, 3, 4, and 5. The longitudinal deformation of the concrete was, therefore, measured on four gage lines at each section, and that of the steel was measured on six bars at sections 1,

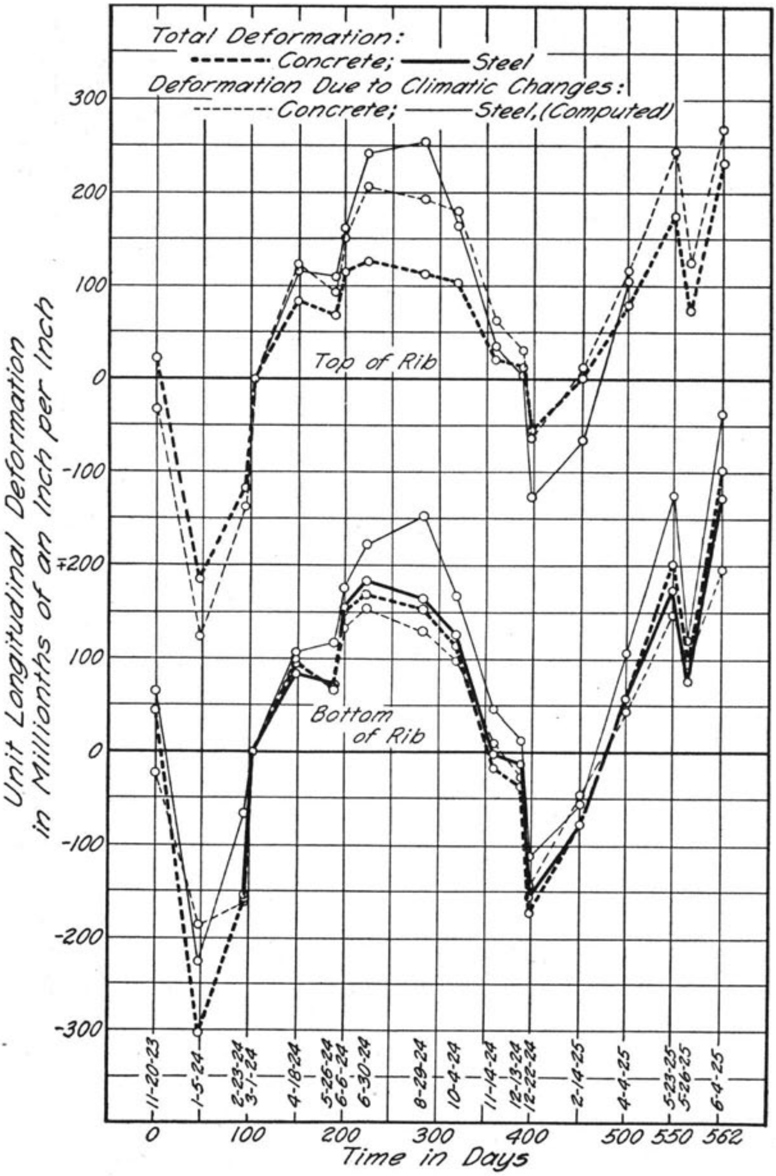

FIG. 10. UNIT LONGITUDINAL DEFORMATION IN RIB, SECTION 1

2, 4, and 5 and on four bars at section 3. The transverse deformation of the concrete was measured on two gage lines, one on the top and one on the bottom of the rib, at sections 1, 2, 4, and 5, and on one gage line on the bottom at section 3.

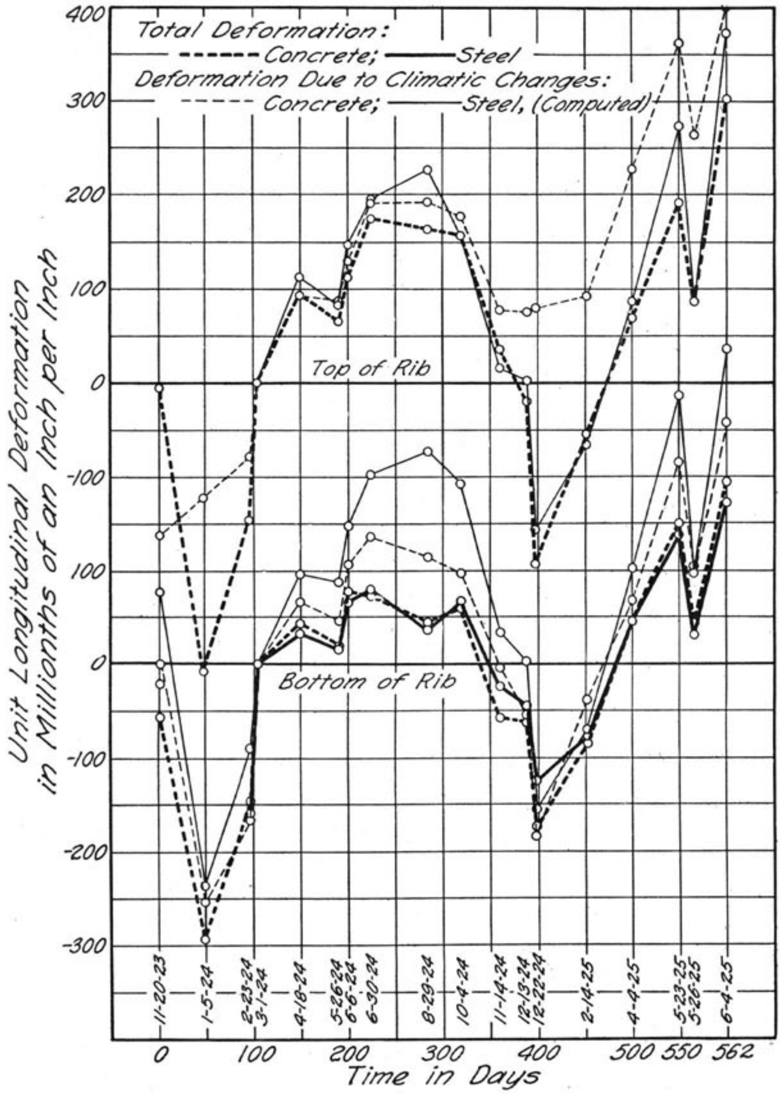

FIG. 11. UNIT LONGITUDINAL DEFORMATION IN RIB, SECTION 2

Two complete sets of strain-gage readings were taken, and the two readings for any point usually checked within 0.0003 inch for the 24-inch gage length; if they did not so check, additional readings were taken until the difference between two successive readings did not ex-

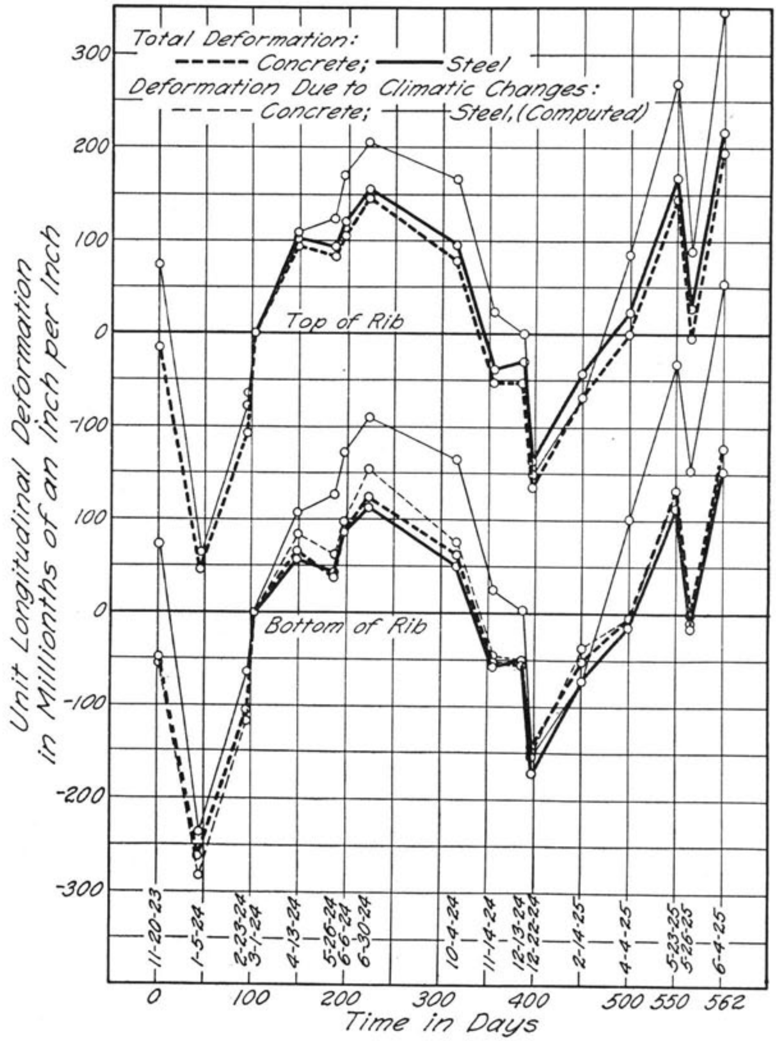

FIG. 12. UNIT LONGITUDINAL DEFORMATION IN RIB, SECTION 3

ceed 0.0003 inch. It was not necessary to take additional readings except in a few cases.

The temperature of the concrete adjacent to the reinforcing bars was measured either just before or just after the strain-gage readings were taken, so that the deformation due to temperature changes could be separated from the deformation due to other causes. The standard

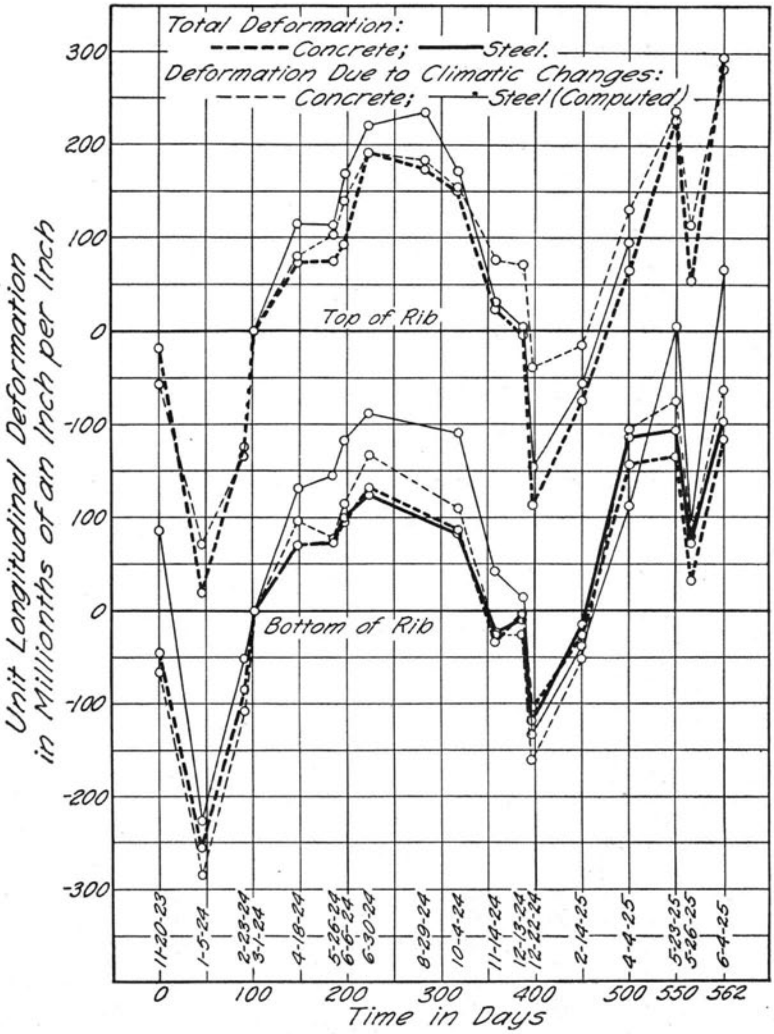

FIG. 13. UNIT LONGITUDINAL DEFORMATION IN RIB, SECTION 4

bar used in connection with the strain gage was a piece of invar steel, having a thermal coefficient of 0.00000065; it was encased in wood $\frac{3}{4}$ inch thick. The temperature of the standard bar was measured with a thermometer incased in the wood in such a way that the bulb fitted into a hole in the steel. All strain-gage readings were adjusted to the readings on the standard bar after the latter had been corrected for changes in its temperature.

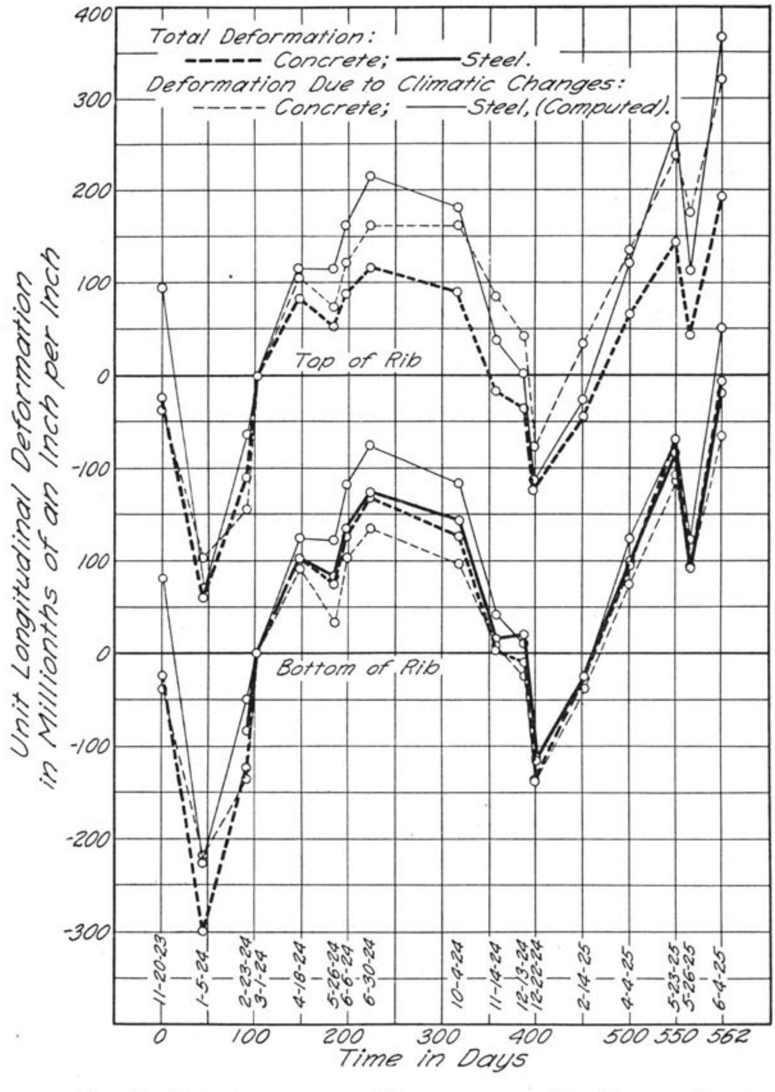

FIG. 14. UNIT LONGITUDINAL DEFORMATION IN RIB, SECTION 5

The total longitudinal unit deformations are presented in Figs. 10 to 14 inclusive, and the transverse unit deformations in Figs. 15 to 20 inclusive. These results are discussed in Sections 11, 12, and 13.

8. *Horizontal Movement of Tops of Piers.*—The tipping of the piers, or the movement of the top relative to the bottom in a direction

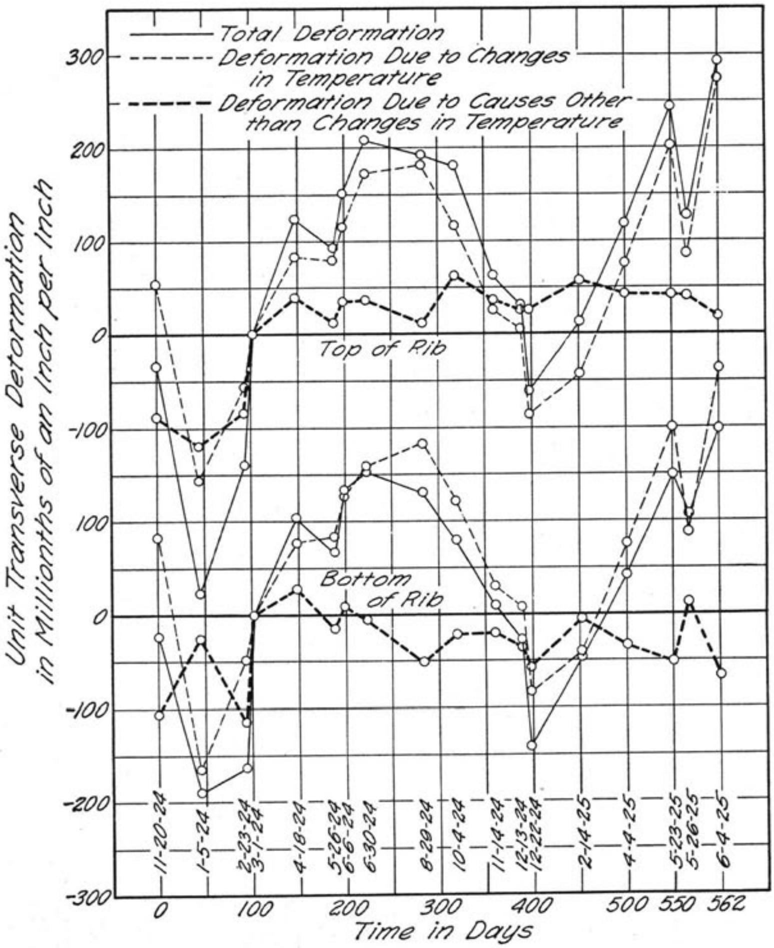

FIG. 15. UNIT TRANSVERSE DEFORMATION IN CONCRETE, SECTION 1

parallel to the roadway of the bridge, was measured with a transit. Steel pins were set in the end of a pier, one near the surface of the ground and the other at the springing of the arch, and a small hole in the end of each pin was used as a target. A transit was set over a bench mark opposite the end of the pier and so located that the line of sight was approximately normal to the axis of the roadway. The transit was set on the target in the upper pin, and then swung down and the offset of the lower target from a vertical line through the upper one was measured with a steel scale graduated to hundredths of an inch. The difference between this offset on any given day and the

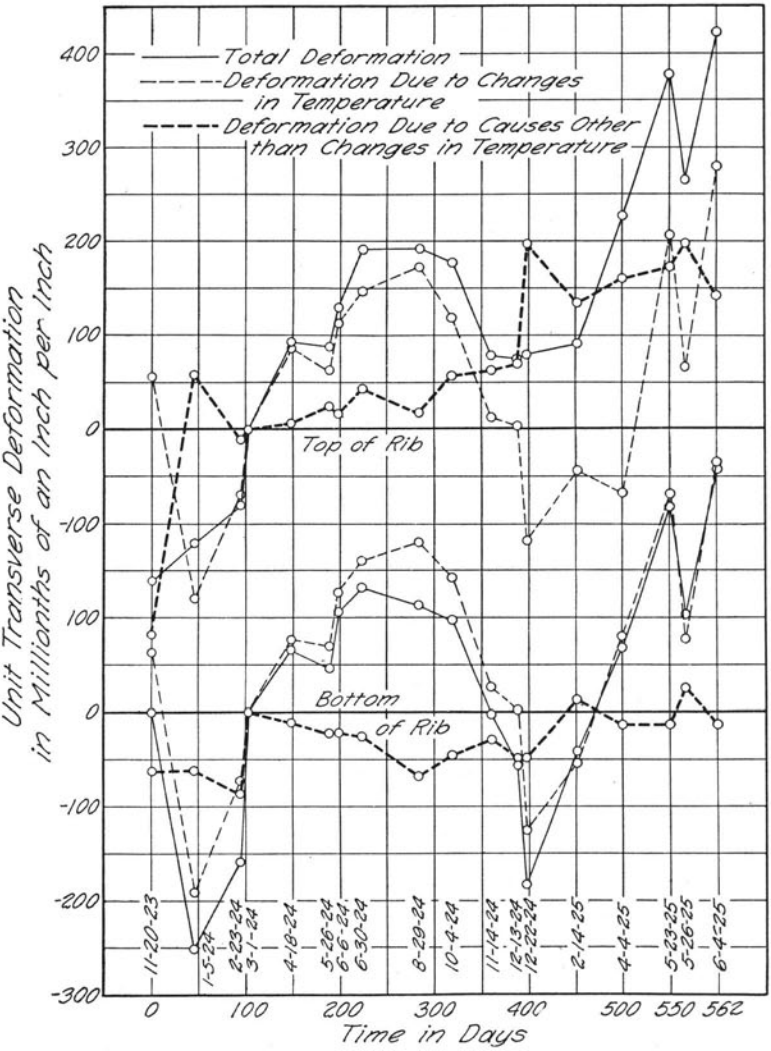

FIG. 16. UNIT TRANSVERSE DEFORMATION IN CONCRETE, SECTION 2

offset on the day when original observations were made, was the amount by which the pier had tipped, or the amount by which the top had moved horizontally with respect to the bottom.

The observed movement of the piers is given in Fig. 21.

Piers 2 and 3, which are at the ends of span 2, tip north or tip south together, there being no variation from this rule except on Dec.

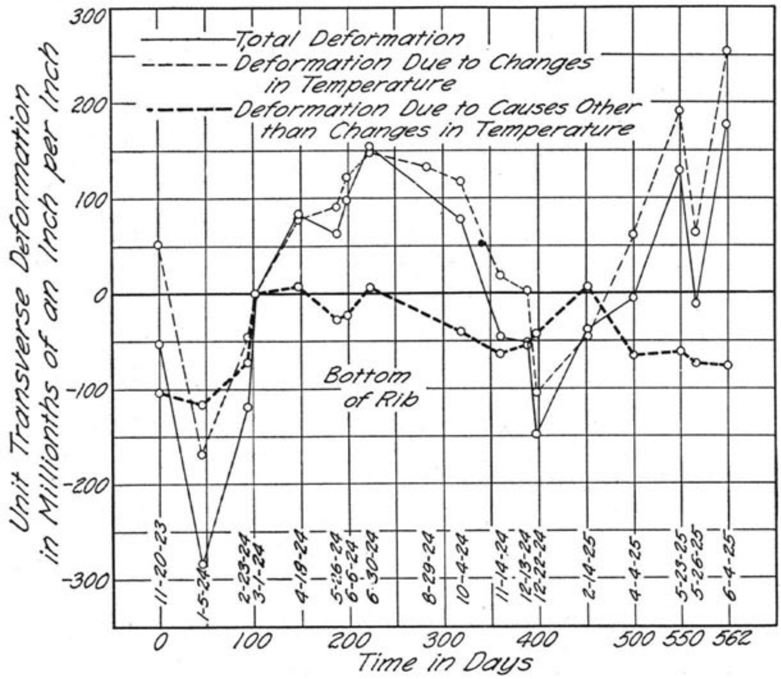

FIG. 17. UNIT TRANSVERSE DEFORMATION IN CONCRETE, SECTION 3

22, 1924.* Fig. 5, page 11, shows that the rotation of the top of a pier, as measured by the levelbar, has the same sense for pier 2 as for pier 3.

It is to be noted that if the distance between the two curves of Fig. 21, one for pier 2 and the other for pier 3, increases, the length of span 2 increases, and if the distance between the curves decreases, the length of the span decreases. Variations in the length of span 2, determined from Fig. 21, are shown in Fig. 22, the full line representing the change in length of the span and the broken line the change in the temperature of the arch rib. This figure is discussed in Section 15.

9. *Change in Width of Expansion Joints.*—The change in the width of the expansion joints was obtained by measuring the distance between centerpunch marks on the ends of steel pins inserted, one on each side of a joint, in the vertical face of the west curb of the roadway. The distance between the punch marks was transferred to a steel scale by means of a pair of machinist's dividers. Duplicate

*Figs. 21 and 22 both indicate that the reading on Dec. 22, 1924 was in error.

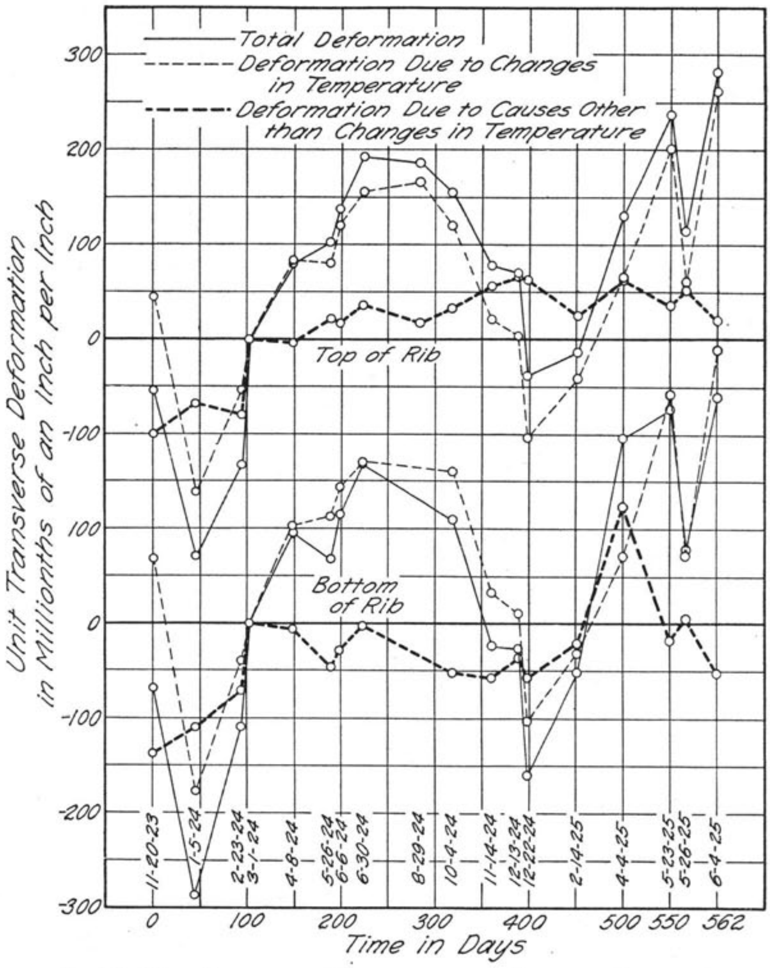

FIG. 18. UNIT TRANSVERSE DEFORMATION IN CONCRETE, SECTION 4

readings were taken, and the two readings for a given point invariably checked within 0.01 inch.

The location of the expansion joints is given in Fig. 1, page 8. The results of the observations are presented graphically in Fig. 23, and they are discussed in Section 16.

III. ANALYSIS OF RESULTS

10. *Change in Temperature.*—The method of measuring the temperature of the concrete is described in Section 4 and the results are given in Fig. 2, page 9.

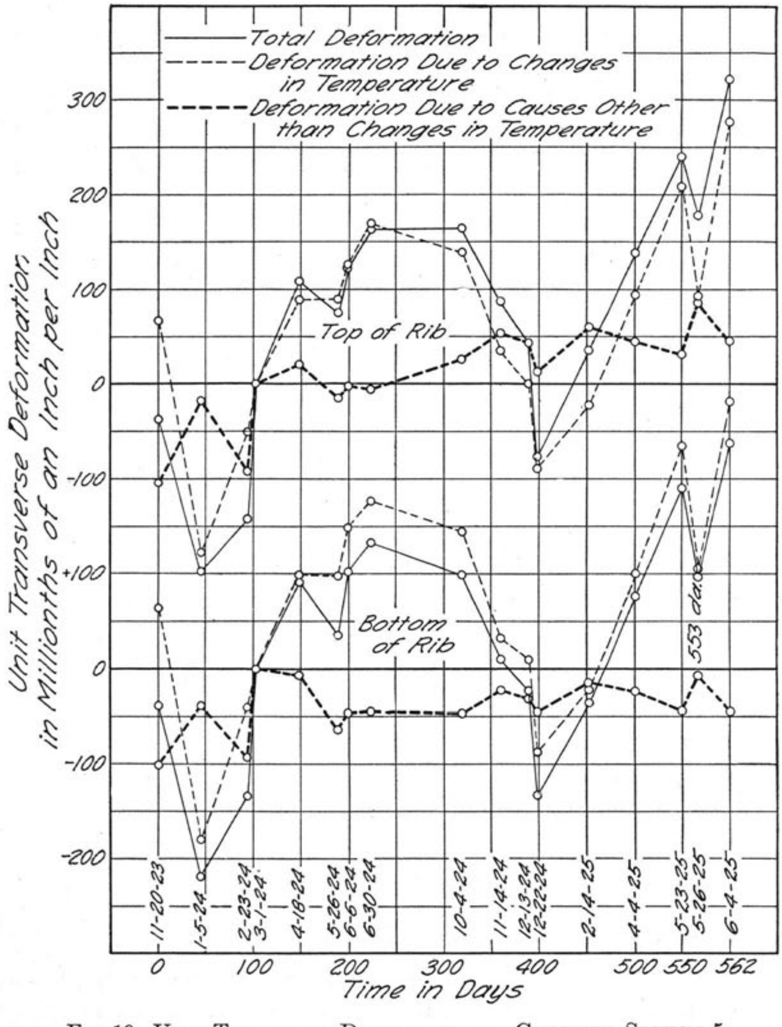

Fig. 19. Unit Transverse Deformation in Concrete, Section 5

The relation between the temperature of the concrete and the temperature of the air is given in Fig. 24. The heavy broken line represents the temperature of the concrete at the center of the section, the heavy full line the temperature 3 inches from the surface, and the light full line the temperature of the air; the first two temperatures are given only for the days when a full set of observations was made, but the temperature of the air is given for each day when observations were made and also for each of the five preceding days in each case.

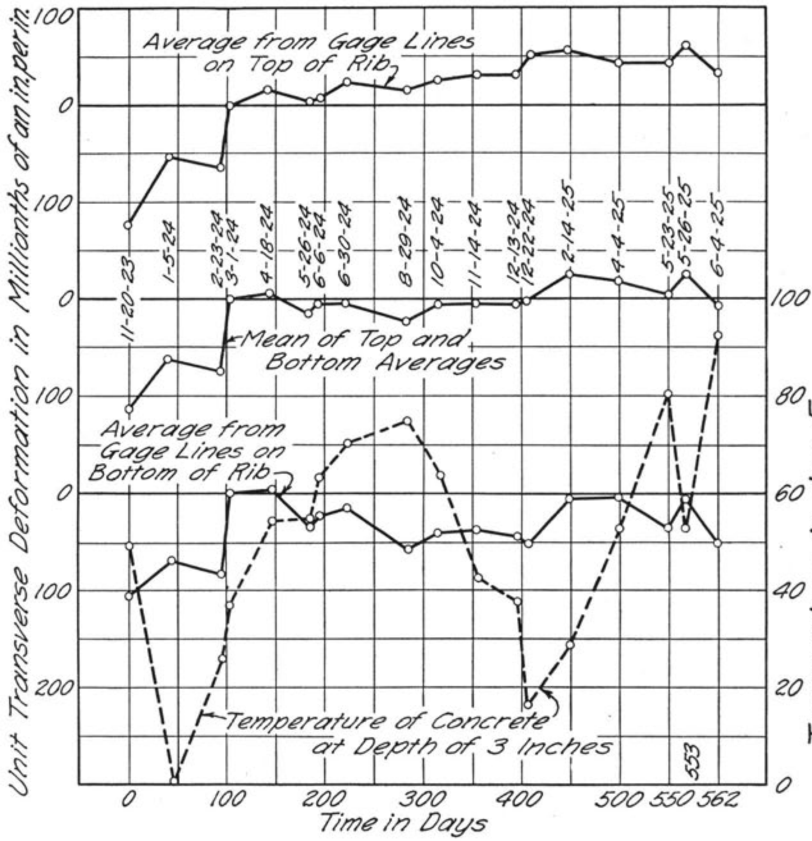

FIG. 20. UNIT TRANSVERSE DEFORMATION OTHER THAN THAT DUE TO TEMPERATURE CHANGES

The temperature of the air given is the mean of the maximum and the minimum temperature for each day, reported by the U. S. Weather Bureau Station at Danville located about a mile from the bridge site.

Variations in the temperature in the 3-inch temperature wells were greater some days than others, the greatest variation, 12.7 deg. F., occurring April 18, 1924, when, according to Fig. 24, the air was colder than the concrete. The lowest temperature on that day occurred at NT1, a point shaded from the sun and exposed to the wind; and the highest temperature occurred at NE6, a point on the east face sheltered from the wind and exposed to the early morning sun. The record of temperatures, given in Table 1, indicates that, except for a few points, the range in temperature, even for this day, was quite small. Table 2 gives the temperatures in the 3-inch wells on June 4, 1925,

TABLE 1

TEMPERATURE OF CONCRETE AT BOTTOM OF 3-INCH TEMPERATURE WELLS*

April 18, 1924

Well No.	Temperature deg. F.	Time when Reading was Taken	Well No.	Temperature deg. F.	Time when Reading was Taken
NT1	50.3	9:00 A.M.	SW5	53.8	11:30 A.M.
NT2	57.0	9:00 A.M.	ST3	56.8	10:30 A.M.
NT3	54.7	9:00 A.M.	ST2	56.0	10:30 A.M.
NW5	51.0	9:00 A.M.	ST1	52.0	10:30 A.M.
NE6	63.0	9:00 A.M.	NB1	50.6	9:00 A.M.
NT7	54.7	2:00 P.M.	NB2	54.2	9:00 A.M.
NT8	55.5	2:00 P.M.	NB3	52.8	9:00 A.M.
NT9	56.0	2:00 P.M.	NB7	51.0	10:00 A.M.
NW11	51.0	10:00 A.M.	NB8	54.4	10:00 A.M.
NE12	53.0	10:00 A.M.	NB9	52.3	10:00 A.M.
CT13	53.6	11:30 A.M.	CB13	54.3	11:30 A.M.
CT14	56.0	11:30 A.M.	CB14	55.0	11:30 A.M.
SE12	54.9	12:00 A.M.	SB9	56.3	12:30 P.M.
SW11	53.2	12:00 A.M.	SB8	56.8	12:30 P.M.
ST9	55.0	12:00 A.M.	SB7	55.2	12:30 P.M.
ST8	55.0	12:00 A.M.	SB3	56.8	10:30 A.M.
ST7	53.6	12:00 A.M.	SB2	56.0	10:30 A.M.
SE6	60.8	11:30 A.M.	SB1	52.5	10:30 A.M.

*The temperature wells can be located on the arch from the following information: N, north; S, south; W, west; E, east; C, center; T, top; and B, bottom.
Numbers 1, 2, 3, 5, and 6 occur at sections 1 and 5; 7, 8, 9, 11, and 12 at sections 2 and 4; and 12, 13, and 14 at section 3. The numbers at a section occur in order from west to east. Thus NT1 is at the west edge and on the top of the rib at section 1; NW5 is on the west face of the same section.

TABLE 2

TEMPERATURE OF CONCRETE AT BOTTOM OF 3-INCH TEMPERATURE WELLS*

June 4, 1925

Well No.	Temperature deg. F.	Time when Reading was Taken	Well No.	Temperature deg. F.	Time when Reading was Taken
NT1	98.1	6:00 P.M.	NB3	91.7	5:30 P.M.
NT2	94.2	6:00 P.M.	NB7	91.5	3:00 P.M.
NT3	94.4	6:00 P.M.	NB8	88.3	3:00 P.M.
NT7	97.2	6:00 P.M.	NB9	90.0	3:00 P.M.
NT8	93.0	6:00 P.M.	SB1	91.0	5:00 P.M.
NT9	93.8	6:00 P.M.	SB2	88.3	5:00 P.M.
CT13	94.3	4:00 P.M.	SB3	91.5	5:00 P.M.
CT14	89.4	3:45 P.M.	SB7	95.3	6:30 P.M.
CB13	94.5	3:45 P.M.	SB8	90.5	6:30 P.M.
CB14	91.0	3:45 P.M.	SB9	91.6	6:30 P.M.
ST9	92.2	5:00 P.M.			
ST8	91.6	5:00 P.M.			
ST7	96.7	5:00 P.M.			
ST3	93.3	5:00 P.M.			
ST2	90.7	5:00 P.M.			
ST1	93.0	5:00 P.M.			
NB1	95.0	5:30 P.M.			
NB2	90.2	5:30 P.M.			

*See note bottom Table 1.

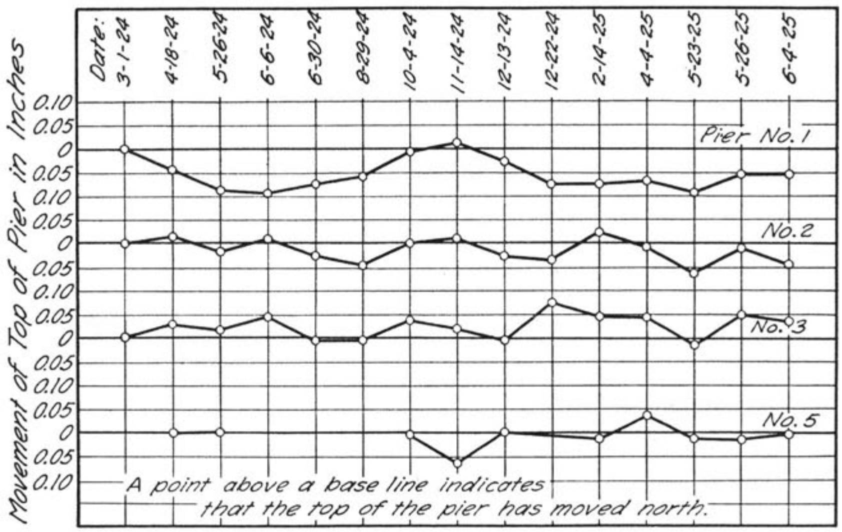

FIG. 21. HORIZONTAL MOVEMENT OF TOPS OF PIERS

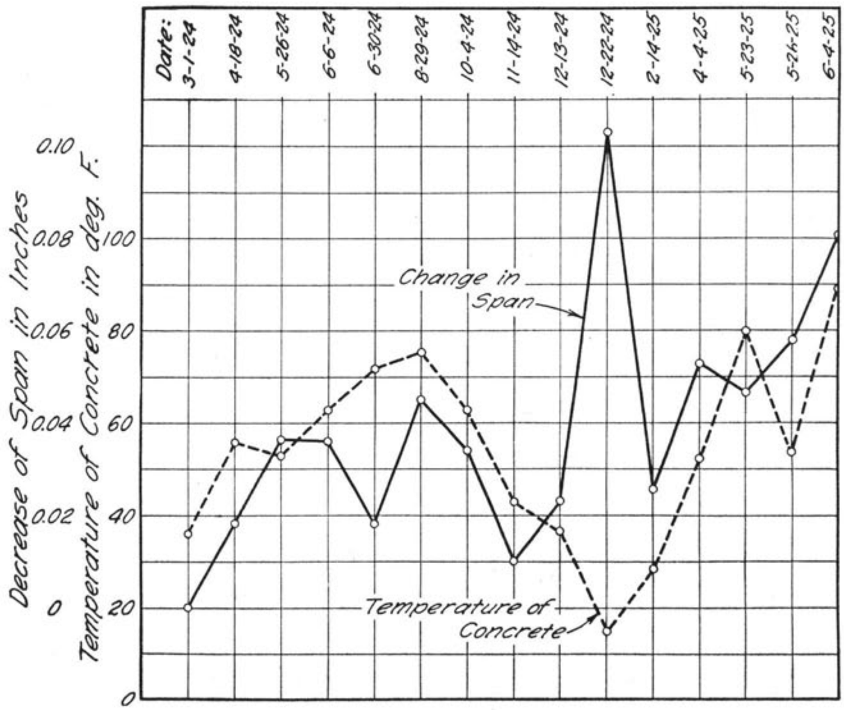

FIG. 22. RELATION BETWEEN CHANGE IN SPAN AND TEMPERATURE OF CONCRETE

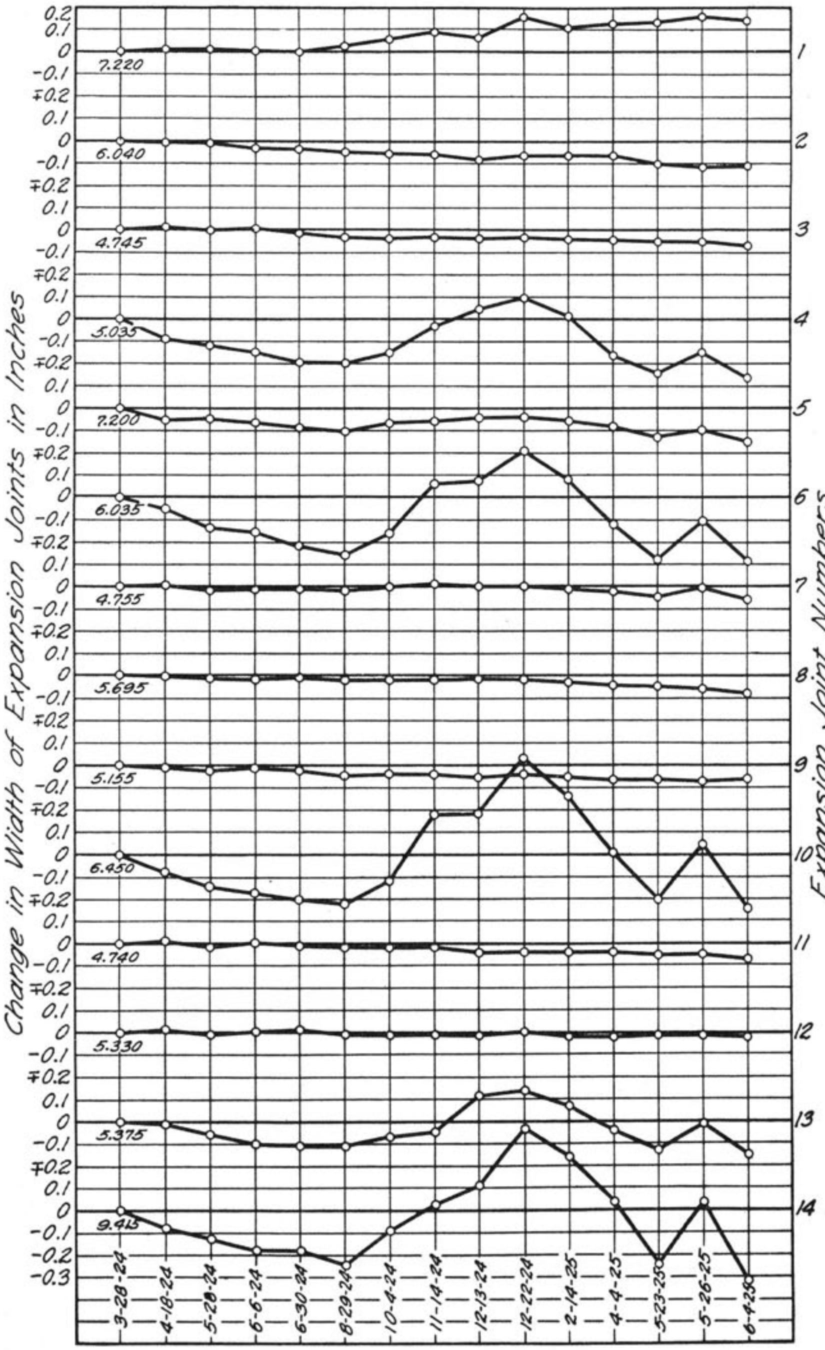

FIG. 23. CHANGE IN WIDTH OF EXPANSION JOINTS

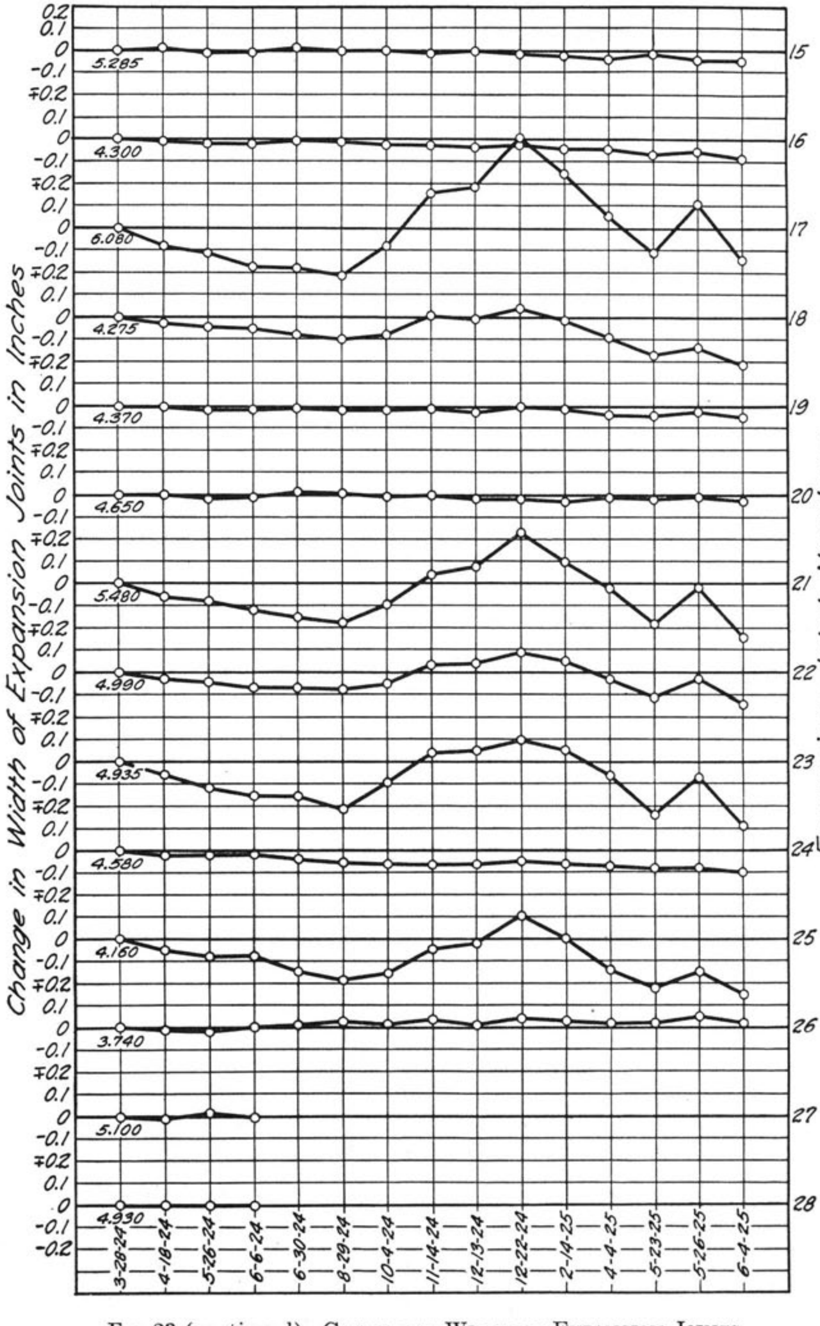

FIG. 23 (continued). CHANGE IN WIDTH OF EXPANSION JOINTS

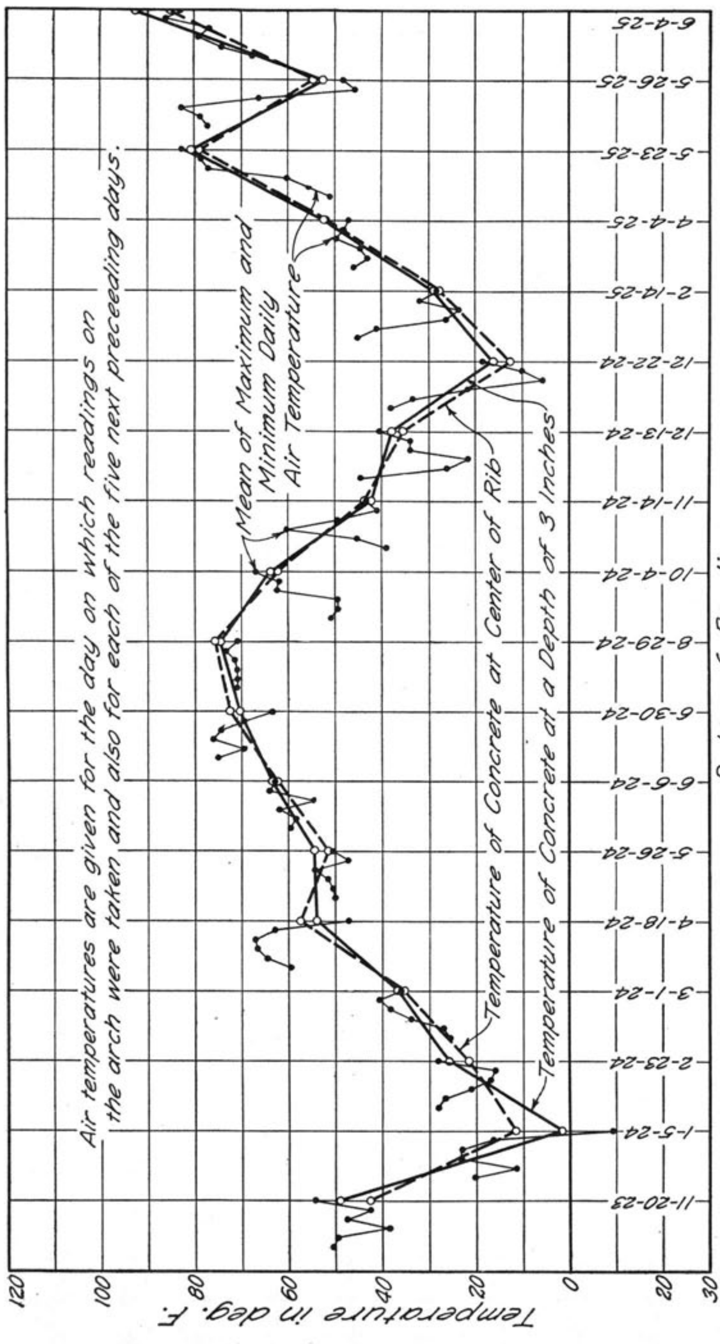

FIG. 24. RELATION OF TEMPERATURE OF AIR TO TEMPERATURE OF CONCRETE

another day on which the range was quite large, being 9.8 deg. F. The lowest temperature was at SB2, the center of the bottom of the rib near its south end, where the sun does not strike except early in the morning and late in the afternoon, and the highest temperature was at NT1, a point on the west side and on top of the rib where the sun had beat directly upon the concrete for several hours before the readings were taken. In general, the variations in the temperature were less than those recorded in Tables 1 and 2. Observations were made on 18 days; for two days the range in temperature at a depth of 3 inches was more than 10 degrees and less than 13 degrees, for 12 days the range was more than 5 degrees and less than 10 degrees, and for 4 days the range was less than 5 degrees. This range in temperature was due, not only to different degrees of shelter, but also to the fact that a period of from 4 hours to 6 hours would elapse between the first and the last readings.

The temperature at the center of a section, following a sudden change in the temperature of the air, varied somewhat with the thickness of the rib. The maximum difference between the temperature at the center where the depth is 40 inches and that where it is 30 inches was 6 degrees; this difference occurred January 5, 1924, just after the sudden drop in the temperature of the air indicated in Fig. 24. On most days, the difference between the two temperatures was much smaller, being more than 3 degrees and less than 6 degrees on 3 days, more than 1 degree and less than 3 degrees on 6 days, and not more than 1 degree on 9 days.

The greatest difference between the average temperature at the center of the rib and that 3 inches from the surface occurred January 5, 1924, just after a very sudden drop in temperature of the air, the difference on that date being 10.2 deg. F. The maximum range in individual readings occurred the same day, the temperature at the center of the 40-inch section being 14.50 deg. F., and at 3-inch well NB1 − 3.5 degrees, the extreme range being 18 deg. F.

Using the mean between the average temperature at the center and at a point 3 inches from the surface, respectively, as the mean temperature of the arch rib, the lowest temperature observed was 6 degrees, recorded January 5, 1924, and the highest was 89 degrees, recorded June 4, 1925, an extreme range of 83 degrees. The cold snap that began the night of January 4-5, 1924, extended through the night of January 5-6 and the arch undoubtedly was colder on January 6 than it was on January 5. Moreover, the mean daily temperature was higher on June 5, 1925, than it was on June 4, the day observations were made. The range of the recorded temperature of the rib, 83 deg. F., was, therefore, less than the maximum range occurring during the period of the observations, and the latter probably had a value of about 90 degrees.

Many engineers believe that the flow of concrete greatly reduces the temperature stresses that would otherwise be developed by the expansion and contraction accompanying changes in temperature. The extent to which the flow of concrete will reduce temperature stresses depends upon the suddenness of the change. The temperature of the rib rose from 14.4 deg. F. on December 22, 1924, to 79.8 deg. F. on May 23, a change of 65.4 degrees in 150 days; it then fell 26 degrees in three days and rose 35.2 degrees during the next nine days.

Figure 24 shows that the temperature of the rib follows the mean daily temperature of the air. This is brought out quite clearly by the records for a number of periods. The average of the mean daily temperatures of the air for the period from December 31 to January 4, 1924, was 18.8 degrees, yet the mean temperature of the rib dropped to 6.4 degrees on January 5 when the mean temperature of the air was — 9.5 degrees; the average of the mean daily temperatures of the air for the period from December 15 to 19, 1924 (the temperatures for December 15 and 16 are not given on the diagram, but were 35.5 and 45.0 degrees, respectively) was 34.3 degrees, yet with mean daily temperatures of the air of 5.5, 10.0, and 18.5 degrees for December 20, 21, and 22, respectively, the mean temperature of the rib on December 22 dropped to 14.4 degrees; and the mean temperature of the rib was 79.8 degrees on May 23, 1925, yet, with mean daily air temperatures of 66.0, 45.5, and 48.0 degrees for May 24, 25, and 26, respectively, the mean temperature of the rib on the 26th was 53.8 degrees, and nine days later it had risen to 89.0 degrees.

On two days, August 29, 1924, and June 4, 1925, the mean temperature of the arch rib was higher than the average of the mean daily air temperatures for the five days next preceding the day on which the complete observations were made. This apparent discrepancy is attributed to the fact that the air temperatures reported were taken in the shade, whereas the arch rib was exposed to the sun in the afternoon. On June 4 the observations on the arch were not completed until 6 p. m., and individual temperatures at a depth of 3 inches were observed as high as 98.1 degrees, yet the mean daily temperature of the air for that day was only 82.5 degrees.

The results of the observations on this bridge indicate that, for an open spandrel arch having a rib from 2 feet to 4 feet deep, the

TABLE 3

THERMAL COEFFICIENT OF CONCRETE AT VARIOUS POINTS ON ARCH RIB

Point on Rib		Thermal Coefficient
Top of Rib	Section 1	0.0000046
	Section 2	0.0000049
	Section 4	0.0000046
	Section 5	0.0000050
	Average	0.00000478
Bottom of Rib	Section 1	0.0000047
	Section 2	0.0000052
	Section 3	0.0000046
	Section 4	0.0000051
	Section 5	0.0000052
	Average	0.00000496
	Mean of Two Averages	0.00000487

maximum probable variation in the mean temperature of the rib is about 90 deg. F. under the climatic conditions existing in central Illinois; and the simultaneous temperatures at various points within the arch probably do not vary by more than 20 deg. F.

11. *Transverse Deformation of Concrete.*—The volume of concrete is affected by stress, by changes in temperature, and by changes in moisture content. The curves of Figs. 15 to 19, pages 20 to 24, show the deformation of the concrete across the rib, a direction in which there is no primary stress. This deformation was measured on a 24-inch gage line, readings being made on both the top and the bottom of the rib at sections 1, 2, 4, and 5, and on the bottom of section 3.

The curves in Figs. 25 to 29, inclusive, show the relation between the deformation of the concrete and the change in its temperature. The arrows on a curve indicate the chronological order of the observations, point 1 in each case representing the conditions on March 1, 1924. The heavy broken line represents the average of all the observations, and the slope of this line is the thermal coefficient of the concrete for the point on the arch corresponding to the figure. The coefficients, as determined for the various points on the rib, are given in Table 3. These coefficients vary from a minimum of 0.0000046 to a maximum of 0.0000052; for each section the coefficient for the top is less than that for the bottom, the average being 0.00000478 for the former and 0.00000496 for the latter; and the mean of the averages is 0.00000487. The minimum individual value is 5.5 per cent less and the maximum is 6.8 per cent greater than the mean of the two averages.

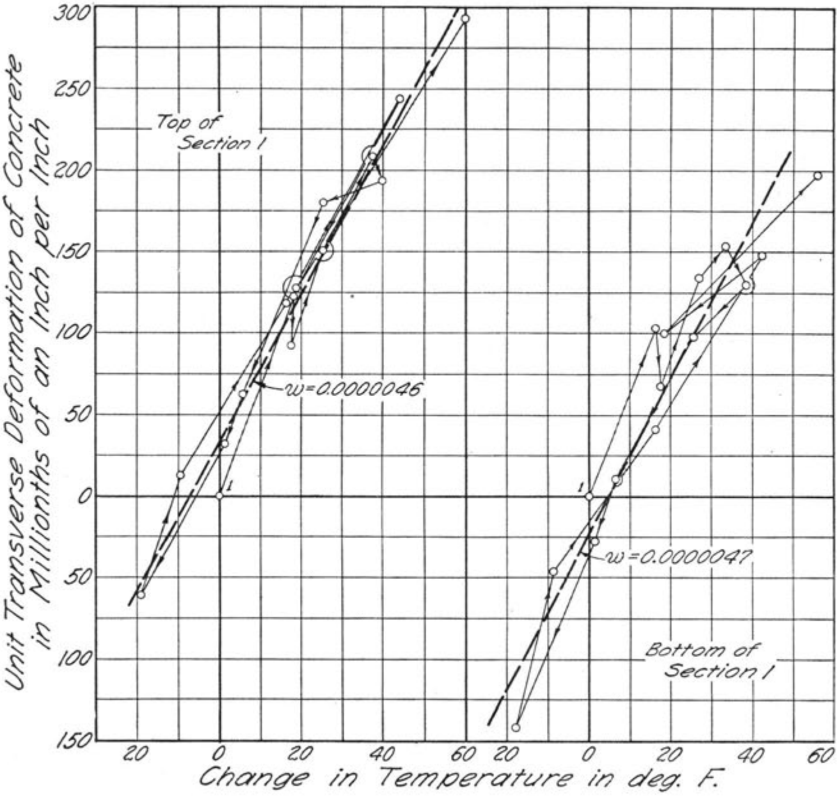

FIG. 25. RELATION BETWEEN VARIATION IN TEMPERATURE AND TRANSVERSE DEFORMATION OF CONCRETE, SECTION 1

The light broken lines of Figs. 15 to 19, pages 20 to 24, represent the computed thermal deformations based upon the observed temperature of the concrete and upon the thermal coefficients given in Table 3; the full lines represent the measured unit deformations of the concrete; and the heavy broken lines represent the difference between the measured and the computed values, the ordinate at any point being the difference between the ordinates of the other two curves for the same point. That is, the ordinates of the heavy broken line represent transverse deformation of the concrete not accounted for by the change in temperature.

The curves of Fig. 20, page 25, represent the average for all sections of that portion of the transverse deformation not accounted for by the change in temperature, the upper curve representing the average

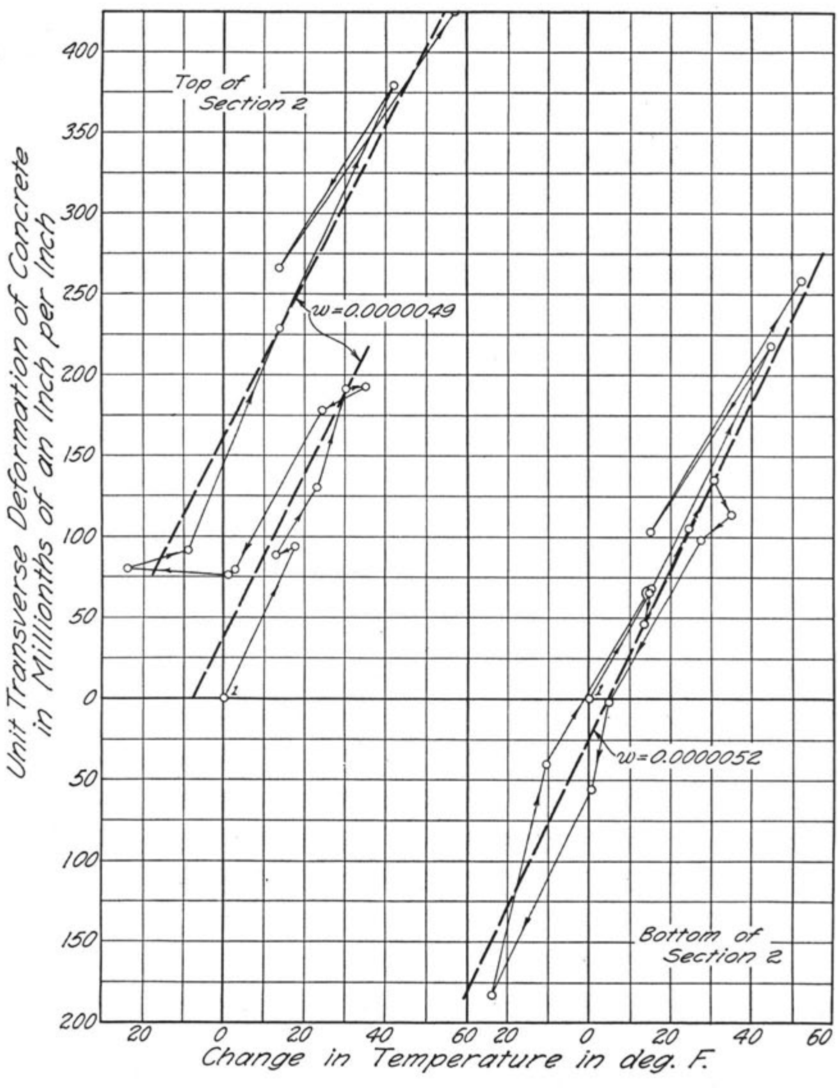

FIG. 26. RELATION BETWEEN VARIATION IN TEMPERATURE AND TRANSVERSE DEFORMATION OF CONCRETE, SECTION 2

value for the top of the rib, the lower one the average value for the bottom of the rib, and the middle one the mean of the other two.

The observations on this bridge indicate that the transverse deformation of the concrete at the individual points does not follow exactly the temperature changes, but that the average for all points

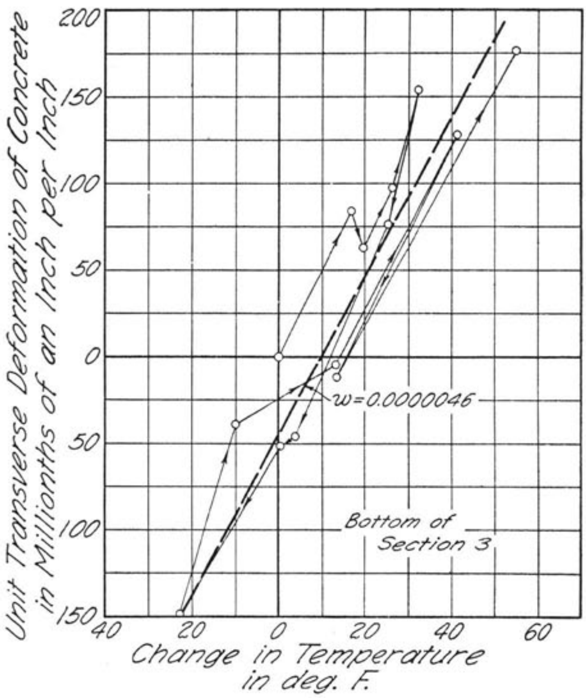

FIG. 27. RELATION BETWEEN VARIATION IN TEMPERATURE AND TRANSVERSE DEFORMATION OF CONCRETE, BOTTOM OF SECTION 3

does, indicating that the transverse deformation, other than thermal, is erratic and is not due to a change in the moisture content of the concrete. The average value of the thermal coefficient of this concrete, which was about three years old at the time the observations were made, is 0.0000049.

12. *Longitudinal Deformation of Arch Rib.*—The total longitudinal deformation of the concrete and steel is given in Figs. 10 to 14, pages 15 to 19. The upper group of curves on each figure gives the deformation for the top of the rib and the lower one that for the bottom. The heavy full lines represent the measured deformation in the steel, and the heavy broken lines the measured deformation in the concrete; the light full lines represent the computed deformation of the steel due to changes in its temperature based upon a thermal coefficient of 0.0000065, and the light broken lines the deformation in the concrete measured on a transverse gage line. The points on the heavy full lines represent the average of six readings, those on the

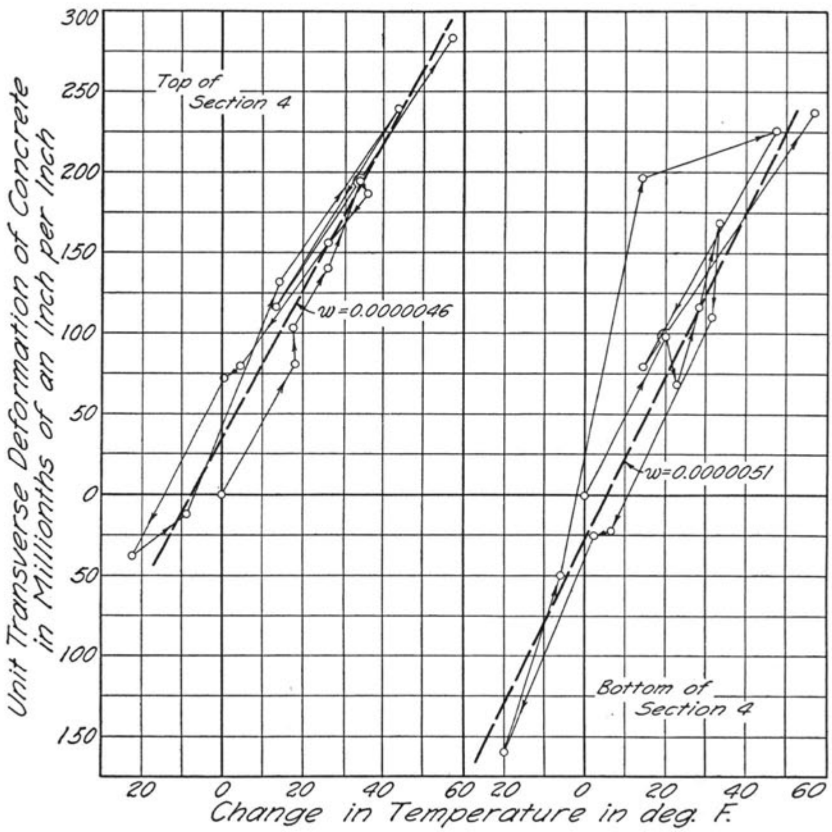

FIG. 28. RELATION BETWEEN VARIATION IN TEMPERATURE AND TRANSVERSE DEFORMATION OF CONCRETE, SECTION 4

light full lines the average of three readings, those on the heavy broken lines the average of four readings, and those on the light broken lines the average of two readings. The light full lines may also be considered as temperature diagrams, since the ordinates vary directly with the temperature of the steel.

The difference between corresponding ordinates of the two full lines represents the deformation due to the stress in the steel, and the difference between the corresponding ordinates of the two broken lines the deformation due to stress in the concrete.* The stress deformation on sections 1, 2, 3, 4, and 5 is given in Figs. 30, 31, and 32 respectively, the heavy full lines representing the unit deformation

*This statement is based upon the assumption that there is no lateral stress deformation.

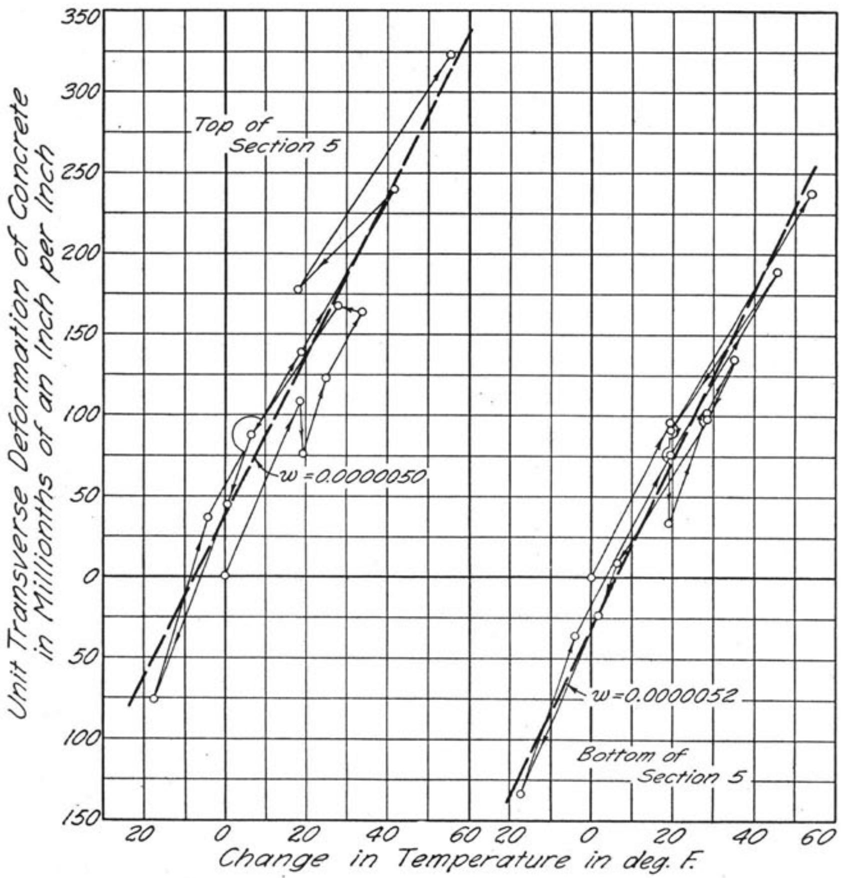

FIG. 29. RELATION BETWEEN VARIATION IN TEMPERATURE AND TRANSVERSE DEFORMATION OF CONCRETE, SECTION 5

due to stress in the steel and the heavy broken lines that due to stress in the concrete.

Strain-gage readings on the steel were taken on both the top and the bottom of the rib but water, collecting in the holes in the concrete on the top of the rib, corroded the steel so that the readings were obviously in error, and were discarded, except at section 3, where the holes in the steel remained in good condition throughout the test. Inasmuch as the deformation in the steel and concrete were approximately the same where both were measured, the total deformation of the concrete (corrected for the difference in the deformation at the surface and at points 3 inches from the surface) was substituted for

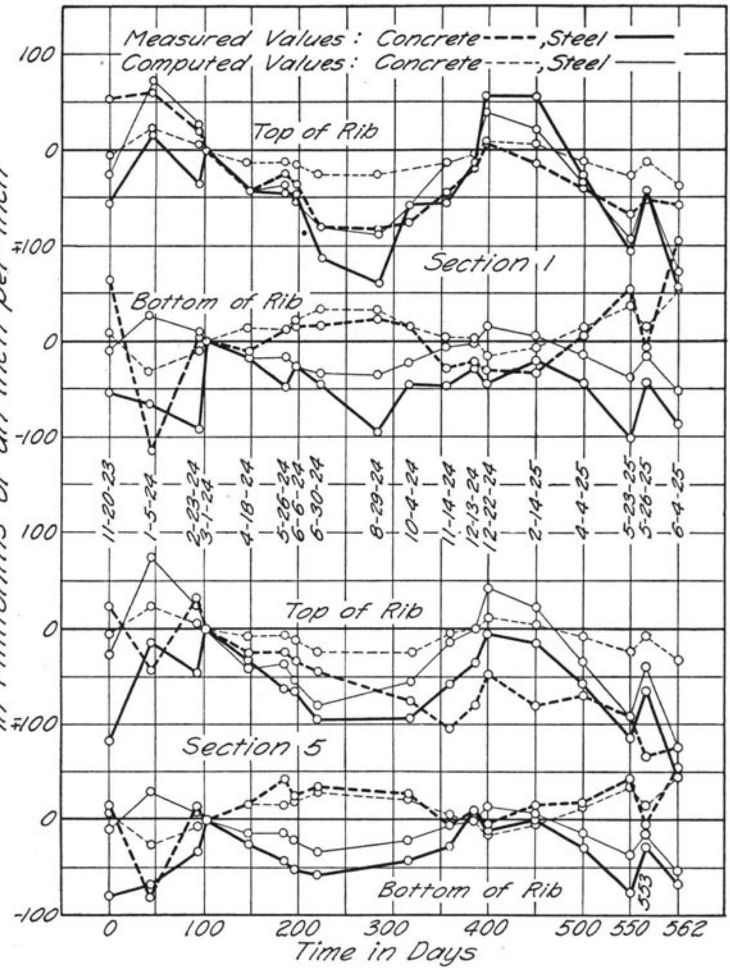

FIG. 30. STRESS DEFORMATION, SECTIONS 1 AND 5

the total deformation in the steel in computing the deformation due to stress in the steel at the top of the rib.

The stress deformation in the concrete, as given in Figs. 30 to 32, was obtained by subtracting the total transverse deformation from the total longitudinal deformation; this procedure is based upon the assumption that there is no transverse stress in the rib and that the Poisson-ratio effect is zero.

The curves of Figs. 30 to 32 show that the measured deformation of the concrete due to stress is greatest at section 2, the unit deforma-

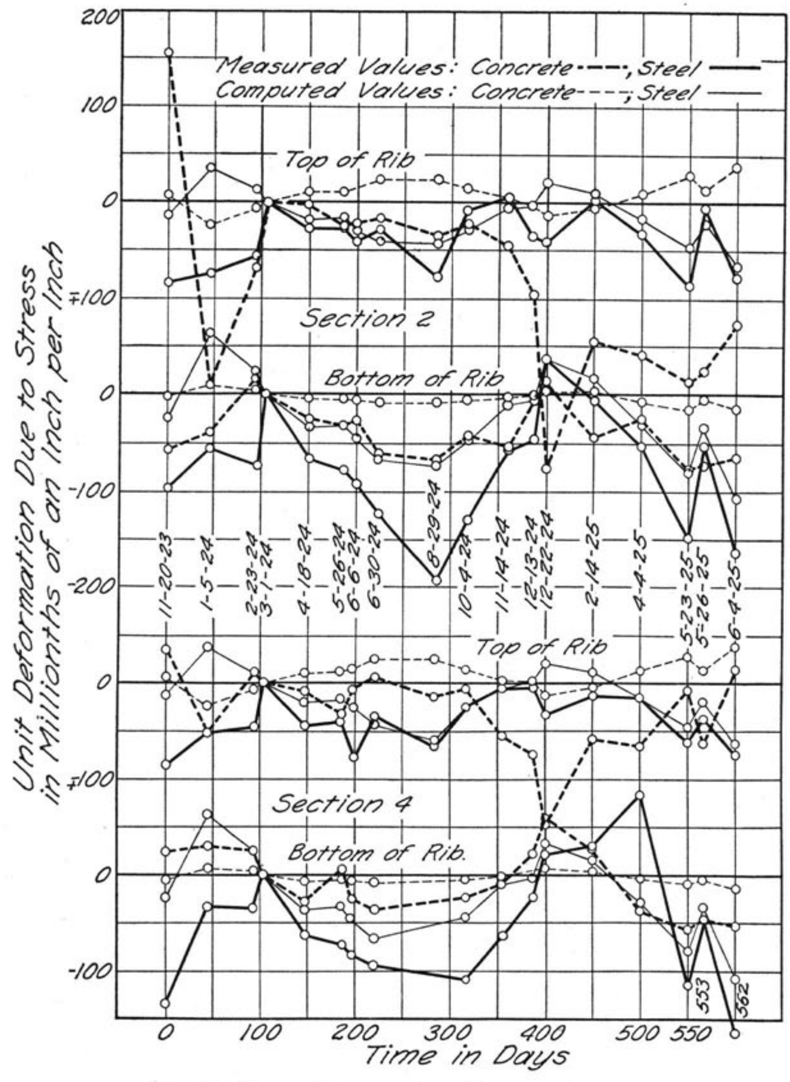

FIG. 31. STRESS DEFORMATION, SECTIONS 2 AND 4

tion at the top of the rib at this section having a maximum range of 0.00024. The compression at the top of the rib at this section accompanying extremely low temperatures is quite large, and the values might be questioned except for the marked similarity of the deformations for the successive winters.

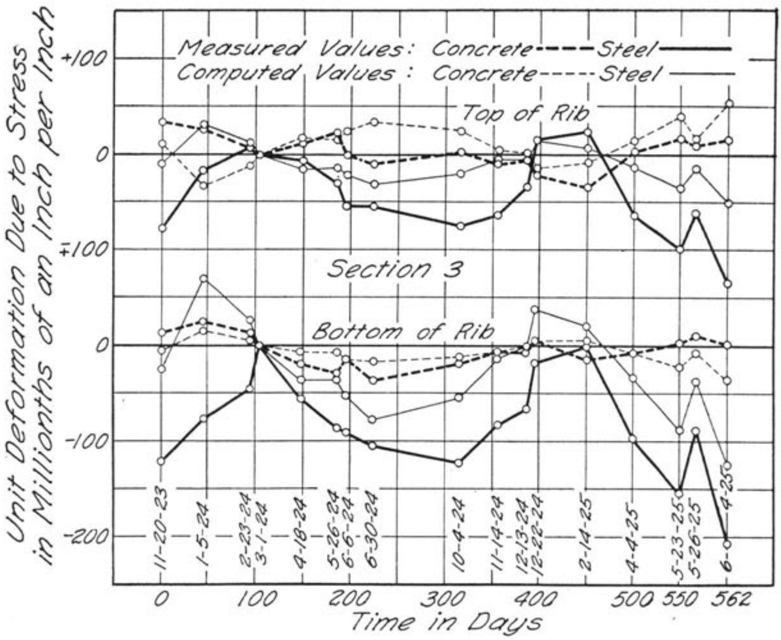

FIG. 32. STRESS DEFORMATION, SECTION 3

The stress deformation is not dependent upon the standard bar readings and, since readings were taken in duplicate and on two gage lines, there is little likelihood of the apparent stress deformation being due to an error in the observations. It is to be noted, however, that the large apparent stress deformation is due partly to the fact that the transverse shrinkage accompanying the drop in temperature was much less than the temperature range should have produced. This is true for January 5, and December 22, 1924, both of the days when readings were taken at extremely low temperatures. Because the transverse deformation at the top of section 2 was not consistent with the temperature change, the stress deformation at the top and bottom of sections 2 and 4 was also determined by subtracting the product of the thermal coefficient and the temperature change at these points from the measured value of the total deformation. The results are presented in Fig. 33, the full lines representing the values based upon the computed thermal deformation and the broken lines the values based upon the measured transverse deformation. The computed thermal deformations are based upon the thermal coefficients for the two sections given in Table 3.

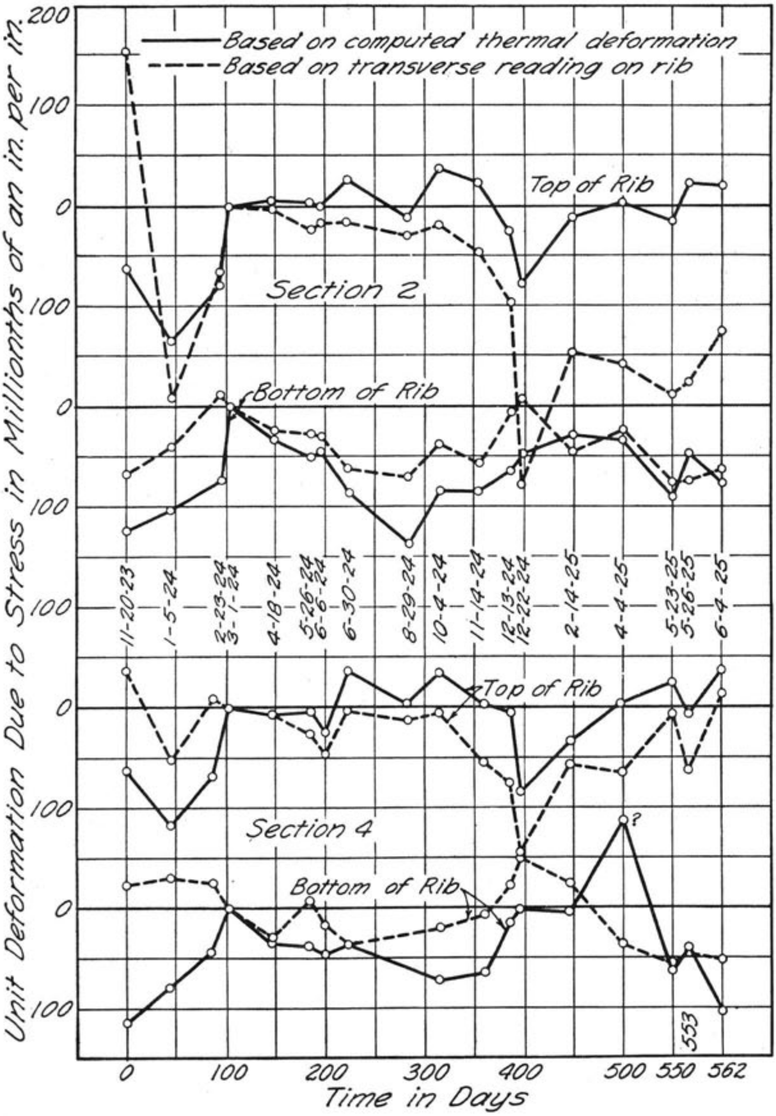

FIG. 33. STRESS DEFORMATION IN CONCRETE, SECTIONS 2 AND 4

The broken-line curves, based upon the transverse deformations, show a maximum range in the unit stress deformation of 0.00024 at the top of section 2; and the full-line curves, based upon the computed thermal deformations, show a range in the unit stress deformation of 0.00017. The method of determining the portion of the deformation that is due to stress by subtracting the transverse from the total longitudinal deformation seems the least likely to be in error, but no explanation is available to account for the very small transverse contraction accompanying the low temperatures of both winters, especially since this small contraction occurred on the top of section 2 and at no other place. Whatever may have been the cause of the discrepancies, there was a range in the unit stress deformation at the top of sections 2 and 4 of at least 0.00017, and the value for section 2 may have been as great as 0.00024. If this stress deformation had been produced instantaneously the required stress would have been, with $E = 2\ 000\ 000$ lb. per sq. in., 340 lb. per sq. in. for the former, or 480 lb. per sq. in. for the latter deformation. Since the deformation was gradual, taking place over a period of approximately six months, the actual stress was probably very much less than these values because of the ameliorating effect of the time yield of the concrete.

The range in the unit stress deformation in the concrete at sections 1 and 5 is from 0.00010 to 0.00015, depending upon the interpretation of the diagrams, and the stress deformation for section 3 is very small, being not over 0.00006. The maximum range in the temperature stress for the steel, given in Figs. 30 to 32 inclusive, is about 6000 lb. per sq. in. The measured stress-deformation diagrams, for both the steel and the concrete, are very similar for corresponding sections at opposite ends of the arch.

The relation between the measured and the computed stress deformation is discussed in Section 13.

13. *Theoretical Temperature-stress Deformation in Rib of Reinforced Concrete Arch.*—The temperature-stress deformation in the rib of a reinforced concrete arch is made up of three parts, as follows: (1) an axial deformation due to the difference between the thermal coefficients of steel and concrete; (2) an axial deformation due to the tangential component of the temperature thrust; and (3) a flexural deformation due to the moment of the temperature thrust. For a given section, the first part of the deformation has the same sense and the same magnitude (approximately) for all points in the concrete and for all points in the steel, but the magnitudes of the deformations for the two materials are different, and the senses are opposite; the second part has the same sense for all sections, and the same sense and the same magnitude for all points on a section, but its magnitude is different on different sections; and the third part has the same sense in the steel as in the adjacent concrete, but the senses on the top and the bottom at a given section are opposite, and the senses on the top of

two sections, one above and the other below the temperature thrust line, are opposite. The resultant of the three parts into which the temperature-stress deformation has been divided may, therefore, be quite different at the top of a section from what it is at the bottom of the same section; it may be different for the steel and for the adjacent concrete, and, in general, it is different at the various sections of the rib.

No mathematical analysis for determining the stress set up in a reinforced concrete block, due to the difference in the thermal coefficients of the two materials, has been universally adopted. The analysis given in the following paragraphs is based upon the assumption that plane sections remain plane. Inasmuch as the stress enters the concrete only where it comes in contact with the steel, the accuracy of this assumption is questionable for short blocks, but seems reasonable for long members such as an arch. The analysis is as follows:

If the temperature rises, the steel has a tendency to expand more than the concrete, but if there is no slip, and if plane sections remain plane, the two must expand together, thus producing tension in the concrete and compression in the steel. For equilibrium, the total tension and the total compression on a given section are equal in magnitude. Moreover, the sum of the unit stress deformation in the concrete and in the steel must equal the difference between the two thermal coefficients. These two statements may be expressed algebraically as follows:

$$A_s f_s = A_c f_c \tag{1}$$

$$\Delta_s + \Delta_c = w_s - w_c \tag{2}$$

in which the notation is as follows:

Δ_s = unit stress deformation in the steel
Δ_c = unit stress deformation in the concrete
w_s = thermal coefficient of steel
w_c = thermal coefficient of concrete
A_s = area of the section of the steel
A_c = area of the section of the concrete
f_s = unit stress in the steel
f_c = unit stress in the concrete
E_s = modulus of elasticity for steel
E_c = modulus of elasticity for concrete

By definition,

$$f_s = \Delta_s E_s$$

$$f_c = \Delta_c E_c \text{ (approximately)}$$

$$p = \frac{A_s}{A_c}$$

$$n = \frac{E_s}{E_c}$$

Combining equations (1) and (2) and simplifying, gives

$$\Delta_s = (w_s - w_c)\left(\frac{1}{1 + pn}\right) \qquad (3)$$

$$\Delta_c = (w_s - w_c)\left(\frac{pn}{1 + pn}\right) \qquad (4)$$

Equations (3) and (4) give the unit deformation in the steel and concrete, respectively, due to the difference between the thermal coefficients for the two materials.

Because this analysis is based upon a questionable assumption, it was checked by a laboratory test as follows:

Two concrete specimens, each 26 inches x 50 inches x 82 inches, were made as nearly alike as possible, except that one had no reinforcement whereas the other contained 12 longitudinal one-inch square bars, 6 near the top and 6 near the bottom of the block. These blocks were stored in the laboratory and attained a temperature of approximately 75 deg. F. Strain-gage readings were taken on the steel and on the concrete and the blocks were then taken out of the laboratory, when they attained a temperature of approximately 20 deg. F., and the strain-gage readings were again recorded. The results reported herewith represent the average of the observations on ten gage lines in the concrete for each block and on eight gage lines on the steel for the reinforced block. A 24-inch strain gage was used, and all readings were checked. The relative deformation of the concrete per degree fall in temperature was 0.00000524 for the the plain block and 0.00000540 for the reinforced one, indicating that the reinforcing bars produced a relative stress deformation of 0.00000016 in the concrete. The relative deformation of the steel per degree fall in temperature, as measured, was 0.00000534. If the thermal coefficient of the steel be taken as 0.0000065, a generally accepted value for steel of this grade, then the unit stress deformation in the steel was 0.00000116. That is, the measured values of Δ_s and Δ_c were 0.00000116 and 0.00000016, respectively.

The value of p, the ratio of the steel to the concrete area, for the reinforced block was 0.00931. The modulus of elasticity of the concrete at zero load, the average of the results obtained from six cylinders, was 3 600 000 lb. per sq. in.; the corresponding value of n is 8.33. Substituting these values for p and n and the values 0.0000065 and 0.00000524 for w_s and w_c, respectively, equations (3) and (4) give

$$\Delta_s = 0.00000117$$

$$\Delta_c = 0.00000009$$

The measured and computed values of Δ_s agree almost exactly; the measured and computed values of Δ_c differ by a large amount, relatively, but the actual difference is insignificant.*

One test is not sufficient to establish definitely the correctness of the analysis that has been presented, but it does give it some support. It is apparent that, with the percentages of steel used in reinforced concrete arches, the stress deformation in the concrete due to the difference between the thermal coefficients for steel and concrete is small; the stress deformation in the steel due to the same cause is appreciable, and its magnitude is directly proportional to the difference between the thermal coefficients of the two materials.

In the application of equations (3) and (4) to the arch at Danville, w_s was taken as 0.0000065 and w_c *as* 0.0000049, the value given in Table 3. The dead load stress in the arch has an appreciable magnitude, so that E should be assigned a value considerably less than the one used in connection with the laboratory test on the blocks; a value of 2 000 000 has been used, making $n = 15$.

The temperature thrust line for span 2, shown in Fig. 4, page 10, is 324.1 inches above the springing, and the horizontal thrust accompanying a one-inch increase in the span is 17 780 lb. The thermal coefficient of the concrete, given in Table 3, is 0.0000049, and the thermal coefficient of structural steel is taken as 0.0000065.

The geometrical properties of sections 1 and 5 are as follows:

Vertical distance of section above springing, 129.7 in.

Width of rib, 60 in.

Thickness of rib, 39.5 in.

Moment of inertia of section, 370 153 in.^{4}

Angle that tangent to the axis of the rib makes with the horizontal, $41° - 20'$.

*For the observed and computed values of Δ_c to be in agreement, the deformation measured on a 24-inch gage line accompanying a 50-degree change in temperature would have to differ from the actual reading by 0.000084 inches, or by a little less than half a division on the Ames dial, a variation within the usually accepted tolerance for the instrument.

Area of steel, 15.1875 sq. in.

Area of concrete equivalent to area of steel, 227.8 sq. in. (E = 2 000 000)

Area of concrete, 2355 sq. in.

Area of concrete equivalent to combined area of steel and concrete, 2583 sq. in.

$p = 0.00645$

Substituting the values of w_s, w_c, n, and p for sections 1 and 5, gives

$$\Delta_s = (0.0000065 - 0.0000049)\frac{1}{1 + (15 \times 0.00645)} = 0.00000146$$

$$\Delta_c = \Delta_s \times pn = 0.00000146 \times 15 \times 0.00645 = 0.000000141$$

in which Δ_s and Δ_c are the unit stress deformations of the steel and of the concrete, respectively, accompanying a change in temperature of one degree F., and due to the difference in the thermal coefficients. If the temperature rises, Δ_s is compression and Δ_c is tension.

The unit deformation in the steel and concrete resulting from the difference in the thermal coefficients of the two materials, due to a rise in temperature of one degree F., is given in columns 7 and 8 of Table 4, for all sections at which strain-gage measurements were taken.

The horizontal thrust resulting from a change in temperature depends upon the thrust produced by a unit change in the span, and also upon the increase in the span that accompanies a unit rise in temperature if the piers are free to move horizontally.

From Fig. 4 a change in span of one inch is seen to produce a horizontal thrust of 17 780 pounds. One degree change in temperature would produce, under free expansion, a change in span of 0.0000049 x 1672 inches, or 0.00819 inch. A rise in temperature of one degree, therefore, produces a thrust of 0.00819 x 17 780, or 145.6 pounds. The tangential thrust at any section equals the product of the horizontal thrust and the cosine of the angle that the tangent to the elastic curve makes with the horizontal. At sections 1 and 5, $\alpha = 41° - 20'$ and the tangential thrust is 109.3 lb. The unit deformation resulting from this thrust is $\frac{109.3}{2\ 583 \times 2\ 000\ 000} = 0.000000021$. If the temperature rises this deformation is compression for both the steel and the concrete.

The unit deformation of the steel and concrete resulting from the thrust accompanying a rise in temperature of one degree F. is

TABLE 4

UNIT STRESS DEFORMATION IN ARCH RIB DUE TO RISE IN TEMPERATURE OF ONE DEGREE FAHRENHEIT

Horizontal thrust due to one degree rise in temperature is 145.6 lb., applied 97.92 inches below crown

Deformations in 0.00000001 inch per inch

Section No.	Unit Deformation Due to Flexure				Unit Deformation Due to Direct Thrust	Unit Deformation Due to Difference in Thermal Coefficients		Total Unit Deformation Due to Temperature Change			
	Top		Bottom					Top		Bottom	
	Steel	Concrete	Steel	Concrete		Steel	Concrete	Steel	Concrete	Steel	Concrete
1 and 5	−64.6	−76.1	+64.6	+76.1	−2.1	−146.0	+14.0	−212.7	−64.2	−83.5	+88.0
2 and 4	+38.0	+48.6	−38.0	−48.6	−3.8	−140.6	+19.4	−106.4	+64.2	−182.4	−33.0
3	+60.9	+78.25	−60.9	−78.25	−4.0	−140.3	+19.7	−83.4	+93.95	−205.2	−62.55

given in column 6 of Table 4 for all sections of the arch on which observations were made.

The flexural stress resulting from the temperature thrust depends upon the magnitude and eccentricity of the thrust, and upon the section modulus of the section being considered. For sections 1 and 5 the moment is $145.6 \times 195.84 = 28\ 514$ in. lb., the moment of inertia of the section is $370\ 153$ in.^{4}, and the half-depth of the rib is 19.75 in. The maximum unit stress in the concrete due to flexure is, therefore,

$$\frac{28\ 514 \times 19.75}{370\ 153} = 1.522 \text{ lb. per sq. in.}$$

The corresponding unit elongation is $\frac{1.522}{2\ 000\ 000} = 0.000000761$ for the concrete and $\frac{19.75 - 2.75}{19.75} \times 0.000000761 = 0.000000646$ for the steel.

For a rise in temperature the deformation at sections 1 and 5 is compression on the top and tension on the bottom of the rib for both the concrete and the steel.

The unit stress deformation in the steel and concrete resulting from the moment in the rib, due to a rise in temperature of one degree F., is given in columns 2, 3, 4, and 5 of Table 4, for all sections at which strain-gage readings were taken.

The resultant stress deformation due to a temperature change is the algebraic sum of the three components into which the deformation has been divided. This resultant deformation, for all sections at which strain-gage readings were taken, is given in columns 9, 10, 11, and 12 of Table 4. The theoretical stress deformation is presented graphically in Figs. 30 to 32, the light broken lines representing the stress deformation in the concrete and the light full lines that in the steel. The heavy lines represent the measured values of the same quantities.

A study of the measured and computed values of the stress deformation, presented in Figs. 30 to 32, leads to the following conclusions:

(1) In general, the measured and the computed values of the deformation were consistent in sense, the two values for the same quantity increasing and decreasing together as the temperature changed.

(2) The measured value and the computed value of the stress in the steel agreed in magnitude remarkably well during the last six months of the test; the measured value exceeded the computed one during the first year of the test by as much as 100 per cent during

the summer of 1924. The greatest discrepancy occurred at the bottom sections 2 and 4, the sections that might be affected by the expansion joints.

(3) The measured and the computed values of the stress deformation in the concrete had approximately the same magnitude at the bottom of sections 1 and 5, and at the top and bottom of section 3; the measured value far exceeded the computed value at the top of sections 1 and 5, and at the top and bottom of sections 2 and 4.

(4) The temperature stress in the steel due to the difference between the thermal coefficients of steel and concrete, exceeded that due to the flexural stress resulting from the temperature thrust.

(5) Some influence, presumably the expansion joints, accentuated the temperature stress in the rib adjacent to the saddle.

14. *Rotation of Piers.*—The method of measuring the rotation of the piers is described in Section 5 and the movements observed are presented in Fig. 5, page 11.

The curves of Fig. 5 show that all of the piers rotated slightly as the temperature changed, the piers for the large central spans rotating more than those for the shorter end ones. Except for one reading, the rotation of pier No. 6 at the south end of the bridge was very small. Piers Nos. 1 and 2 at the north end rotated an appreciable amount but not as much as Nos. 3, 4, and 5.

The most remarkable feature in connection with the rotation is the fact that, with a rising temperature, the piers north of the longest span tipped south and those south of that span tipped north. That is, the piers tipped toward the long span with a rising temperature. This tendency is especially clear for that portion of the curves that shows the results obtained from the three last observations, May 23, May 26, and June 4, 1925, when there was an abrupt drop and then an equally abrupt rise in the temperature.

The magnitude and position of the horizontal thrust due to a change in the temperature of the concrete of one degree, based upon the elastic theory of arches (the spandrel columns and deck being neglected), are given in Fig. 4. This shows that, as the temperature rises, the large span, No. 4, should tip piers 4 and 5 outwards; likewise, spans Nos. 3, 2, and 1 should tip piers 3, 2, and 1 north; and span No. 5 should tip pier No. 6 south. Instead, although piers 1 and 6 have little rotation, piers 2, 3, 4, and 5 have tipped in a direction just the opposite from that in which they should have tipped if they were controlled by the thrusts of Fig. 4.

TABLE 5

NOTES ON EXPANSION JOINTS

Joint on Side Toward Long Span			Joint on Side Away from Long Span		
Condition of Joint	Joint No.	Maximum Movement at Joint	Condition of Joint	Joint No.	Maximum Movement at Joint
		Pier 1			
Width of opening in concrete, 0.38 in. Filler had fallen out except at curb.	4	0.36 in.	Width of opening in concrete, 0.31 in. Crack tightly filled with filler.	3	0.08 in.
		Pier 2			
Width of opening in concrete, 0.75 in. Filler loose. Joint open at top of hand rail.	6	0.50 in.	Width of opening in concrete, 0.5 in. Nearly all of the filler had fallen out of joint in sidewalls. Joint tight at top of hand rail.	5	0.15 in.
		Pier 3			
Width of opening in concrete, 1.75 in. Filler had all fallen out.	10	0.67 in.	Width of opening in concrete, 0.06 in. Crack tightly filled with filler.	9	0.08 in.
		Pier 4			
Width of opening in concrete, 2 in. Filler had all fallen out.	14	0.68 in.	Width of opening in concrete, 0.56 in. Loose filler in opening.	13	0.29 in.
		Pier 5			
Width of opening in concrete, 2 in. Filler had all fallen out.	17	0.62 in.	Width of opening in concrete, 0.19 in. Crack tightly filled with filler.	18	0.25 in.
		Pier 6			
Width of opening in concrete, 0.75 in. A little loose filler in place.	21	0.47 in.	Width of opening in concrete, 0.5 in. Crack tightly filled with filler.	22	0.22 in.
		Pier 7			
Width of opening in concrete, 0.75 in. A little loose filler in place.	23	0.39 in.	Width of opening in concrete, 0.12 in. Crack tightly filled with filler.	24	0.10 in.

The change in width of the expansion joints affords a check upon the rotation of the piers as measured by the levelbar. Referring to Fig. 1, the expansion joints at pier 4 are 13 and 14, and at pier 5 they are 17 and 18. Figure 23, page 28, shows that, with a rising temperature, joint 14 closed much more than joint 13, indicating that pier 4 had tipped south; likewise, with a rising temperature, joint 17 closed much more than joint 18, indicating that pier 5 had tipped north. Both of these movements are in accord with the levelbar readings presented in Fig. 5, and they are contrary to the temperature thrusts of Fig. 4. The movement of the other expansion joints also confirmed the sense of the rotation of the other piers as determined by the levelbar.

The most plausible explanation of the contrary rotation of the piers seems to be that the deck slab is in contact with the extension of the piers on the side toward the shore, so that, as the temperature rises, the deck expands, thus tipping the piers toward the center of the bridge. This conclusion is based on the information contained in Table 5.

The action of the bridge was apparently as follows: The deck expanded with a rise in temperature and, since it butted against the city pavement at the end of the bridge, moved away from the shore. The expansion joint on the shore side of the pier was closed, causing the deck to exert an inward thrust at the top of the first pier. The expansion joint on the channel side of the pier was open, thus preventing the pier from transmitting the thrust to the adjacent deck, and submitting the pier to an unbalanced overturning thrust at the level of the deck. As a result of this unbalanced thrust, the pier tipped toward the channel, moving the rib inward relative to the deck, thus producing a shear in the spandrel columns. This shear forced the deck in the second span against the next pier, and, since the joint on the shore side was closed and the one on the channel side open, this pier, likewise, tipped toward the channel. The action was continued toward the channel, spanned by the longest rib, from each end of the bridge. Since, for the long span, both piers had open joints on the side toward that span, both had unbalanced thrusts at the elevation of the deck that tended to make the pier tip inward as the temperature rose, thus actually shortening the span. The author believes that this is the true explanation of the peculiar behavior of the piers of this bridge. There is no doubt that the piers did tip toward the long span as the temperature rose, and that the expansion joints adjacent to the piers were, in general, open on the side of the pier toward the long span and closed on the side away from that span.

Some explanation is necessary to account for the expansion joints on the shore side of the piers being closed while those on the channel side were open an undue amount. The plans of the bridge call for expansion joints of equal width on both sides of the piers, and no doubt the joints were of equal width when the bridge was built. Apparently, therefore, the deck had either moved inward relative to the piers or the piers had moved outward relative to the deck. The expansion joints originally were filled with asphalt one inch thick and where, at a given pier, the asphalt had been squeezed out of the joint on the shore side, the width of the channel side joint was invariably greater than the original, and the sum of the two joints was as great as ever. It is apparent, therefore, that the piers had tipped out rather than that the deck had crept in. The following explanation is offered to account for the movement of the piers:

The pressure diagrams of Fig. 7 show that the unit soil pressure on the footings of the piers due to dead load is not uniform, the maximum on the shore side being twice as great as the minimum on the channel side for some of the piers. The maximum unit pressure would not be considered excessive for the grade of shale underlying these piers, providing the pressure were uniformly distributed. But, with the variation in pressure shown, the settlement would not be uniform over the entire area and the pier would tip.* The distance from the base of the pier to the deck is from 4 to 5 times the width of the base. The maximum movement of the top of the pier, as indicated by the closing of the expansion joint on the shore side of the piers, probably did not exceed one inch, a movement accounted for by an unequal settlement of one-quarter of an inch over a width of about 20 feet, a value that could reasonably be expected with the pressure variation noted.

The dead load thrusts, given in Fig. 7, are for arches fixed at the springing. The elastic deformation of the piers would slightly reduce the unbalanced moment; on the other hand, a rise in temperature would produce a thrust that would increase the unbalanced moment. These two influences, both of which have been neglected, would offset each other leaving the total unbalanced moment due to causes continuously effective over a considerable period of time substantially in accord with the values shown in the figure.

The rotation of the piers due to the unequal soil pressure is of interest because of the stress it produces in the ribs. The maximum probable movement at the top of the deck is one inch and the dis-

*The dead load soil pressure is quite uniform over the sub-base of pier 5, nevertheless, this pier tipped. The tipping may have been due to the big variation in pressure on the top surface of this sub-base.

tance from the bottom of the footings to the top of the deck is approximately 90 feet. The angle of rotation, θ, would therefore equal $1 \div 1080 = 0.00093$ radian. The largest stress will be produced at the springing of span 4 since, for it, both piers tipped outward. The thickness of the rib at the springing of span 4 is given in Fig. 4 as 5.33 feet, and the moment produced at the springing due to a rotation of 0.001 radian is given in Fig. 7, as 6 476 201 in. lb. at the end rotated and 2 892 041 in. lb. at the opposite end. The maximum moment produced, if both piers tip out 0.0093 radian, is 0.93 (6 476 201 + 2 892 041) = 8 712 465 in. lb. The rotation of the piers has a further effect in that it increases the length of the span. Assuming the center of rotation to be at the bottom of the footing, the motion at the springing would be about one-half as great as at the deck, or 0.5 inch for each pier. If both piers tip outward, as they did in the case of span 4, the tipping of the piers might increase the length of the span one inch. From Fig. 7 it is seen that by increasing the length of the span one inch a moment at the springing of 11 204 190 in. lb. is produced. This added to the moment resulting from the change in the angle gives a total moment of 19 916 655 in. lb.

The section is 64 inches thick, 72 inches wide, and has a moment of inertia equal to 1 850 000 in.[4] The stress resulting from the rotation is, therefore,

$$p = \frac{19\ 916\ 655 \times 32}{1\ 850\ 000} = 344 \text{ lb. per sq. in.}$$

These computations are based upon a value of E of 2 000 000 lb. per sq. in. and upon the assumption that the deformation was produced quickly. As a matter of fact the deformation extended over a long period of time and the time yield of the concrete greatly relieved the stress. The fact should also be borne in mind that the computations are based upon a movement of the pier at the level of the deck equal to one inch, a maximum possible rather than a probable value. Nevertheless, this motion of the piers, resulting from an unequal distribution of the soil pressure, produced an accidental deformation too large to be ignored.

The contrary tipping of the piers accompanying a change in temperature is of interest because of its possible effect upon the span length and the corresponding effect upon the temperature stress. Piers 2 and 3, being on the same side of the long span, rotated in the same direction; since the rotations were of approximately equal magnitude they did not materially affect the length of span 2, the extreme variation, shown in Fig. 22, being 0.08 inch, which is about one-eighth of the free thermal expansion of a span 139 feet 4 inches long. The change in span, therefore, increased the temperature stress about 12½ per cent in span 2. Both piers for span 4 tipped in as the temperature rose, thus shortening the span. Unfortunately, readings were not taken to determine the horizontal movement of the tops of these piers. Estimates based upon the rotation of the piers, given in Fig. 5, indicate that the maximum shortening of the span could not exceed one-half of the free expansion, indicating that the horizontal movement of the top of the piers could not increase the temperature stress more than 50 per cent.

Figure 7 shows that a rotation of pier 3 of 0.0001 radian produces a moment in the rib of span 2 equal to 287 586 in. lb. The maximum observed rotation, 0.00036 radian, would produce a unit stress at the springing of 38 lb. per sq. in. with E at 2 000 000 lb. per sq. in. This stress is of no particular significance, and as the rotation of the other piers is not materially greater, the stress resulting from the rotation of the piers due to temperature changes does not seem to be great enough to deserve special consideration.

The observations on this bridge have led to the following conclusions relative to the rotation of the piers accompanying changes in the temperature of the concrete:

(1) The piers rotated sufficiently for the movement to be measured accurately, but the rotation was not great enough for the direct effect to be serious except possibly in span 4.

(2) The rotation of the piers of this bridge was opposite in sense from what would be expected from the unbalanced temperature thrust of the ribs alone.

(3) The uneven distribution of the dead load pressure on the pier footings probably caused the piers to rotate until their tops were in contact with the deck.

(4) The rotation of the piers in a sense opposite to that which the temperature thrust of the rib should produce was due to the deck being in contact with the piers on the side away from the large span, causing the piers to move in and out as the deck expanded and contracted.

(5) The temperature stress in span 2 was increased about 12½ per cent by the contrary rotation of the piers, and that in span 4 may have been increased a greater amount, but probably not more than 50 per cent.

15. *Rise and Fall of Crown.*—The method of measuring the rise and fall of points on the arch rib is described in section 6 and the results of the observations are given in Fig. 8.

The crown is the only point on an arch for which the theoretical rise and fall are usually computed. The average of the movements of points N6 and S6 has been taken as the movement of the crown. This movement is presented in Fig. 9, the broken line representing the rise and fall of the crown, and the full line the temperature of the concrete. The ordinates for the two diagrams were arbitrarily chosen so that the distance that represents one degree change in temperature also represents 0.01 inch vertical movement of the crown. It so happens that, with this relation between the scales, the two diagrams almost coincide. This indicates that one degree change in temperature produces a vertical movement of the crown of approximately 0.01 inch.

The theoretical rise and fall of the crown is given by the equation

$$\Delta h = tLw\left(i_h + \frac{h}{L}\right)^*$$

in which $w = 0.0000049$, $L = 1672$ inches, $h = 423.25$ inches, and i_h, the mid-ordinate of the influence diagram for the thrust at the crown $= 0.998$.

Substituting these values in the equation, with t equal to unity, gives

$$\Delta h = 0.0102$$

as the theoretical movement accompanying a one-degree change in temperature, based upon the assumption that the abutments are fixed.

Figure 22 shows that a rise in temperature of one degree decreases the span approximately 0.0008 inch. This movement of the piers should increase the vertical movement of the crown $0.998 \times 0.0008 =$ 0.0008, which, added to 0.0102, gives, as the theoretical movement, 0.011 inch, compared with 0.01 inch, the actual movement as measured.

The relation between the temperature and the vertical movement of the crown was very constant except for the period from Nov. 14 to Dec. 13, 1924. During this period, although the temperature of the arch fell 6.3 degrees, the crown rose a little over 0.04 inch, and it retained this increment in its elevation throughout the remaining period of the observations.

*"Temperature Deformations in Concrete Arches," by Hardy Cross, Engineering News-Record, Feb. 4, 1926, p. 190.

The question naturally arises whether the "jump" in the diagram was due to an actual change in the position of the crown or to an error in the observations. An error in the observations could be occasioned in two ways, (1) a change in the tape, or (2) an error in individual readings. The only portion of the tape that would affect the elevation of the crown relative to the springing is the portion that is used in taking readings at N6 and S6, and that is not used at N1. This eliminates the end of the tape and the ring. The only alteration in the body of the tape that would affect the readings would be a break and an inaccurate splice. No breaks occurred, and as the same tape was used for all readings the possibility of alterations in the tape affecting the readings seems to be eliminated. Relative to errors in the individual readings, although mistakes in individual readings do occur, the probability of the same error occurring for both the original and the check readings on eight successive days is extremely remote. The author is convinced that Fig. 9 represents the actual movement of the crown.

Change in the length of the arch would affect the elevation of its crown. The change in span was computed from the horizontal displacement of the tops of the adjacent piers, measured with a transit in the manner described in Section 8. The relation between the change in span and the temperature of the rib is shown in Fig. 22, the full line representing the changes in the length of the span, and the broken line the temperature changes. It is to be noted that, in general, the span decreased as the temperature increased and increased as the temperature decreased. An exception to this rule is the decrease in the span for the period from Nov. 14 to Dec. 13, 1924, when the temperature of the rib decreased 6.3 degrees. This contrary decrease in the span would account for the contrary rise in the crown, shown in Fig. 9, during the same period. It should be noted, however, that Fig. 22 contains other discrepancies relative to the temperature of the concrete and the length of the span that are not reflected in the relation between the temperature of the concrete and the position of the crown.

The change in the width of the expansion joints affords a check upon the change in length of the span as determined by the transit. In general, the expansion joints should open with a falling and close with a rising temperature. An examination of the diagrams for expansion joints Nos. 6 and 9 in Fig. 23 shows that the movement of these joints was consistent with the change in temperature for all of the periods except for that from Nov. 14 to Dec. 13, 1924. For the latter period, there is no change in the width of any of the expansion joints between piers 2 and 3. Since the deck contracted with a drop in temperature, and since none of the expansion joints functioned, the piers must have been pulled together, thus shortening the span and raising the crown as shown in Fig. 9.

Movement at the expansion joints is accompanied by a sliding of the deck upon its support. It would not be surprising if this motion occurred in jumps instead of gradually as the temperature changed.

There is a disagreement as to the correct expression for the rise and fall of the crown of an arch due to changes in temperature of the concrete. The equations used in computing the theoretical value $h = tLw\left(i_h + \frac{h}{L}\right)$* considers the vertical movement to be made up of two parts, as follows: (1) the portion $tLwi_h$, due to the piers being restrained against horizontal motion (this portion of the movement of the crown produces bending in the rib); (2) the portion htw, the free expansion in a vertical direction of a height of rib equal to h (this portion of the movement produces no stress in the rib). Most text books on arches neglect the latter part and give, for the vertical movement of an arch rib, the equation $h = tLwi_h$. The observations on span 2 support the equation $h = tLw\left(i_h + \frac{h}{L}\right)$.

16. *Change in Width of Expansion Joints.*—The method of measuring the width of the expansion joints is described in Section 4 and the results are presented in Fig. 23.

The most interesting feature in connection with the movement at the expansion joints is the fact that the joints at the saddle did not change in width. The failure of these joints to function might be due to two causes, as follows:

(1) A change in temperature of the concrete causes an expansion or contraction that should have produced motion at the joint, but, because of a defect in the construction or some other accidental cause, no motion occurred. Inasmuch as there was no appreciable motion at any of the eight expansion joints adjacent to the saddle, and since it is hardly likely that they would all be defective, this does not seem to be a plausible explanation.

(2) The rotation of the points where the spandrel columns join the rib causes the deck to move longitudinally, relative to the saddle, as the temperature changes. This phenomenon is explained in detail in the following paragraph.

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*See footnote p. 56.

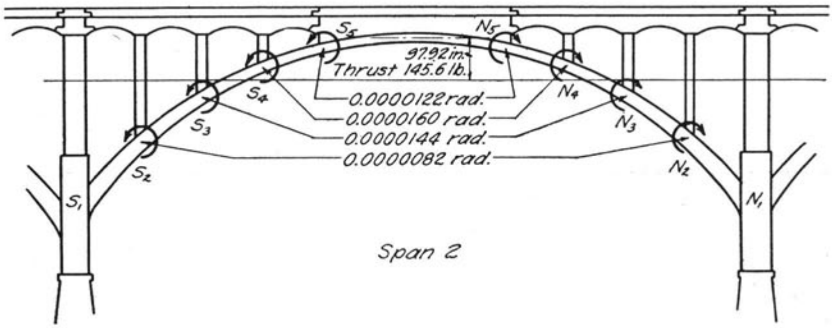

FIG. 34. ROTATION OF POINTS ON ARCH RIB DUE TO A ONE-DEGREE RISE IN TEMPERATURE OF CONCRETE

Figure 34 shows the rib, spandrel columns, deck, and saddle of span 2. There are expansion joints at 6, 7, 8, and 9, those at 7 and 8 being adjacent to the saddle. If the temperature of the concrete rises one degree, the pier being fixed, the effect upon the arch is the same as would be produced by moving the tops of the piers in 0.0082 inch without permitting them to rotate. This motion of the piers rotates every point on the rib, the sense of the rotation being indicated by the arrows in the figure.

As the temperature rises, the rotation of the bases of the spandrel columns will move the deck away from the saddle. At the same time the expansion of the deck causes both of its ends to move away from its center. The effects of the rotation of the bases of the columns and of the expansion of the deck are opposite in sense at the end of the deck adjacent to the saddle, and have the same sense at the piers. If the magnitude of the two components of the movement were equal there would be no movement at the joints at the saddle and the expansion of the deck over the whole span would be provided for by the expansion joints at the piers.

The magnitude of the rotation shown in Fig. 34 is the theoretical value based upon the elastic properties of the rib, the effect of the deck being neglected. The measured value of the rotation at the various points was obtained from the data given in Fig. 6 by applying the method used in Figs. 25 to 29 to obtain the thermal coefficient of concrete. The measured and the computed values are compared in Table 6. The rotation of the piers had some effect upon the rotation of points on the arch rib. Both the measured and the computed values of the rotation indicate that the rotation of points on the rib will cause the deck to move away from the saddle as the temperature rises.

TABLE 6

COMPARISON OF MEASURED AND COMPUTED VALUES OF ROTATION OF POINTS ON ARCH RIB

Rotation is in radians per degree F. rise in temperature

Point	Measured Value	Computed Value
N1	−0.0000036	0.0
S1	+0.0000040	0.0
Av	+0.0000002	0.0
N2	+0.0000114	+0.0000082
S2	+0.0000140	+0.0000082
Av	+0.0000127	+0.0000082
N3	+0.0000137	+0.0000144
S3	+0.0000120	+0.0000144
Av	+0.0000129	+0.0000144
N4	+0.0000128	+0.0000160
S4	+0.0000080	+0.0000160
Av	+0.0000104	+0.0000160
N5	+0.0000049	+0.0000122
S5	0.0	+0.0000122
Av	+0.0000025	+0.0000122

The exact magnitude of the movement of the deck, resulting from the rotation of points on the rib at the bases of the columns, is difficult to determine because of the complexity of the structure. The following approximate analysis is, however, of interest.

The purpose of the expansion joint at the saddle is to relieve the shear in the spandrel columns. The rigidity of the deck is so great relative to the rigidity of the column that the top of the column is held in a vertical position.

If the spandrel column is free from shear its action is similar to that of a cantilever subjected to the action of a couple at its outer end. It can be proved that the deflection of the cantilever from a tangent at the support is given by the equation $d = 0.5\ h \times \theta$, in which d is the deflection, h is the length, and θ is the slope at the end where the couple is applied.

If the expansion joint at the saddle is omitted the ideal condition exists when the expansion of the deck from the saddle to the top of any column equals $0.5\ h\ \theta$ for that column, in which θ is the rotation of the base of the column and h is its height.

The expansion of the portion of the deck between the saddle and each of the spandrel columns is compared with the deflection of the columns in Table 7, the movements being due to a rise or fall in the temperature of 50 deg. F. from a mean. The expansion of the deck between the saddle and Columns N2, N3, and N4 is given in Columns 5, 8, and 11, respectively of the table; the deflection of Columns N2,

TABLE 7

CALCULATED MOVEMENT AT EXPANSION JOINTS ADJACENT TO SADDLE DUE TO TEMPERATURE CHANGE OF 50 DEG. F. EACH WAY FROM MEAN

How θ Was Determined	Values of θ			Based Upon Expansion from Saddle to Column at Point								
				N2 or S2 . h = 323 in.			N3 or S3 . h = 212 in.			N4 or S4 . h = 132 in.		
	$\theta_2 = 0.5\ (\theta_{N2} + \theta_{S2})$	$\theta_3 = 0.5\ (\theta_{N3} + \theta_{S3})$	$\theta_4 = 0.5\ (\theta_{N4} + \theta_{S4})$	Expansion of Deck Between Saddle and N2. 50 x 0.0000056 x 440	Movement at Top of Column N2 (or S2) Due to θ_2 50 x 0.5$h\theta_2$*	Theoretical Movement at Expansion Joint (5) − (6)*	Expansion of Deck Between Saddle and N3. 50 x 0.0000056 x 288	Movement at Top of Column N3 (or S3) Due to θ_3 50 x 0.5$h\theta_3$*	Theoretical Movement at Expansion Joint (8) − (9)*	Expansion of Deck Between Saddle and N4. 50 x 0.0000056 x 136	Movement at Top of Column N4 (or S4) Due to θ_4 50 x 0.5$h\theta_4$*	Theoretical Movement at Expansion Joint (11) − (12)*
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)
Computed	0.0000082	0.0000144	0.0000160	0.1232	0.0662	±0.0570	0.0806	0.0763	±0.0043	0.0381	0.0528	∓0.0147
Measured	0.0000127	0.0000129	0.0000104	0.1232	0.1025	±0.0206	0.0806	0.0684	±0.0122	0.0381	0.0343	±0.0038

*This is based upon the assumption that the rib from N5 to S5 does not bend, or that $\theta_5 = 0$

N3, and N4 is given in Columns 6, 9, and 12, respectively; and the difference, or the theoretical movement at the expansion joints, is given in Columns 7, 10, and 13. The quantities in the first line of the table are based upon the computed values of θ given in Table 6; those in the second line are based upon the measured values of θ, also given in Table 6.

The values given in Table 7 show that, for a rise or fall of 50 deg. F., if there is no expansion joint, a maximum expansion or contraction of 0.057 inch remains unprovided for and can only be relieved by an axial deformation of the deck or a flexural deformation of the spandrel column at N2. This value depends upon a theoretical value of θ based upon the assumption that the spandrel columns and deck do not affect the flexure of the rib. The greatest expansion unprovided for, if the measured values of θ are used in the computations, is 0.02 inch. The fact should also be noted that if computed values of θ are used, the temperature deformation from the saddle to N2, not compensated for by the rotation of the base of the column at N2, is opposite in sign to the similar deformation from the saddle to N4. In other words, assuming an expansion joint at the saddle, if the action of the structure out to N4 causes the joint to open, the simultaneous action out to N2 will cause it to close. That is, a joint would only partially relieve the stress resulting from a temperature change. The maximum expansion to be provided for is only 0.057 inch, and this can be transferred to the expansion joint adjacent to the piers if the spandrel column at N2, which is 25 feet high and 2 feet wide, is deflected 0.057 inch.

The θ's were measured near the axis of the rib and the measured values probably should be used in the analysis. If the measured values of θ are used, the maximum expansion to be provided for is only 0.02 inch and this also can be transferred to the joint at the piers if the column 25 feet high and 2 feet thick is deflected 0.02 inch.

The approximate quantitative analysis of the structure agrees, therefore, with the observations on the expansion joints, and it seems to be quite satisfactorily established that it is unnecessary to provide expansion joints adjacent to the saddle of span 2.

The relation between the motion caused by the direct expansion of the deck and that due to the rotation of the bases of the spandrel columns depends upon the shape of the rib and upon the distance from the crown of the rib to the deck. The observations show that the proportions of the Gilbert Street Bridge were such that expansion joints were not needed adjacent to the saddle for any of the spans.

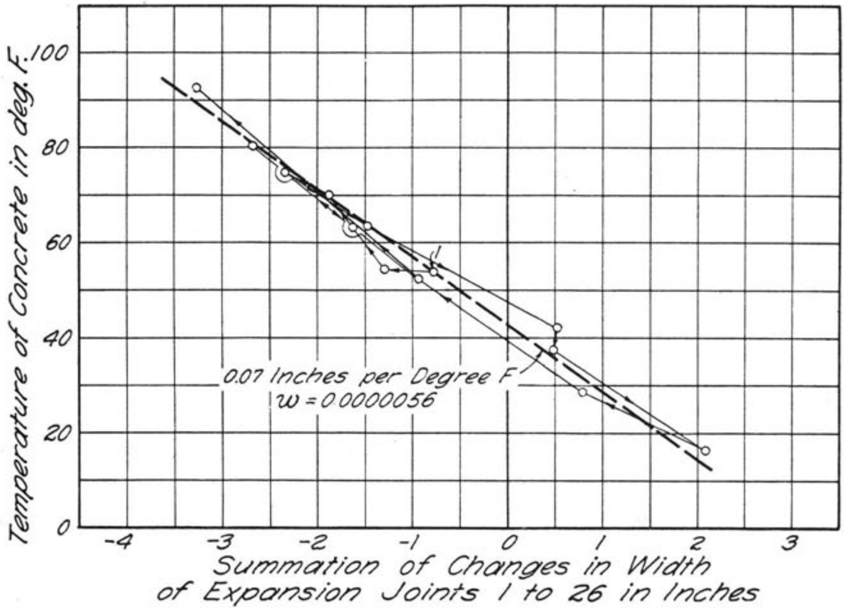

FIG. 35. RELATION BETWEEN CHANGE IN TEMPERATURE OF CONCRETE AND MOVEMENT AT EXPANSION JOINTS

The movements at joints 1 to 26 inclusive, all of the joints from the city pavement on one side of the bridge to that on the other side, were added together and their sum plotted against temperatures. The resulting diagram is shown in Fig. 35 in which the ordinates represent temperatures of the concrete and the abscissas represent the sum of the movements of all of the joints. The arrows indicate the chronological relation of the points.

The temperature in the rib at a depth of 3 inches was used as the temperature of the deck, as no readings were taken of the latter. Inasmuch as the expansion of the deck was invariably read at the end of a set of observations, in many cases after sundown, and always so late in the afternoon that the curb and sidewalk were shaded by the railing on the bridge, it is believed that the temperature of the deck was not appreciably different from the temperature of the rib.

The maximum range in the sum of the movements of all expansion joints was from + 2.09 inches on Dec. 22, 1924, to − 3.28 inches on June 4, 1925, a total of 5.37 inches. This expansion and contraction occurred on a total length of bridge of 1046 feet. The slope of the broken line in Fig. 35 represents the total expansion of the deck per degree change in temperature, and the quotient of this slope divided by the length of the deck over which the expansion takes place, is the thermal coefficient of the deck, in this case 0.0000056. This is slightly more than the thermal coefficient of the concrete in the rib, given in Table 3. It is interesting to note that the total movement of the expansion joints agrees so well with the total free expansion of the deck.

IV. Summary

17. *Summary of Conclusions.*—The conclusions resulting from the observations on the Gilbert Street Bridge at Danville, Illinois, may be summarized as follows:

(1) There was an observed range in the mean temperature of the arch rib of 83 deg. F. The maximum range that occurred during the period of the observations was probably about 90 degrees.

(2) The most rapid change in the mean temperature of the rib was a drop of 26 degrees in three days followed by a rise of 35.2 degrees in the succeeding nine days.

(3) The crown rose and fell with changes in the temperature, the relation between the movement and the temperature change being very constant and almost exactly (within 10 per cent) equal to the theoretical relation. The maximum range in the vertical movement was 0.62 inch.

(4) The thermal coefficient of the concrete was 0.0000049 for the rib and 0.0000056 for the deck.

(5) The change in the temperature caused the piers to rotate but the movement was not of sufficient magnitude to produce appreciable stresses.

(6) The dead load thrusts produced an unequal distribution of the soil pressure on the footings of the piers and caused the piers to rotate until they came in contact with the deck on the side away from the long span.

(7) There was practically no movement at the expansion joints adjacent to the saddles. The sum of the movements of all the expansion joints was equal to the theoretical thermal expansion of a concrete slab extending from the pavement in the city streets at one end of the bridge to that at the other. The sum of the movements at all of these joints had a maximum value of 5.37 inches.

(8) The measured value of the temperature-stress deformation, in both the steel and the concrete, was greater than the com-

puted value at all sections, but the difference was not great except at sections 2 and 4; at these sections the measured value was very much greater than the computed value. The large temperature-stress deformation at these two sections was attributed to the abrupt change in the effective section at the end of the saddle. The rib, saddle, and deck apparently functioned as one solid piece, whereas the deck and rib outside of the expansion joints adjacent to the saddle did not so function, there being an appreciable relative horizontal motion between the two.

(9) The expansion joints adjacent to the saddle are not needed to relieve the temperature stress in the deck and they seem to weaken the structure by localizing the temperature deformation in the rib at the ends of the saddle. If these expansion joints had been omitted the bridge would have been cheaper, the roadway would have been smoother, and, in the opinion of the author, the structure would have been stronger.

(10) The soil pressure due to dead load should be uniformly distributed over the footing, for an uneven distribution of the pressure may cause the pier to tip even though the maximum pressure is no greater than would be permitted if the pressure were uniform.

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